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SB—117—2022

FACULTY OF SCIENCE

B.Sc. (First Year) (First Semester) EXAMINATION

MAY/JUNE, 2022

(New Course)

MATHEMATICS

Paper II

(Algebra and Trigonometry)

(Saturday, 18-06-2022) **Time : 10.00 a.m. to 12.30 p.m.**

Time— 2½ *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. (A) (*i*) Prove that the necessary and sufficient condition for a square matrix A to possess the inverse in that $|A| \neq 0$ *i.e.* A is non-singular. 8

(*ii*) Find the inverse of the matrix : 7

$$A = \begin{bmatrix} 9 & 5 & 6 \\ 7 & -1 & 8 \\ 3 & 4 & 2 \end{bmatrix}$$

Or

(B) Define the following : 8

(*i*) Minor of order k of a matrix.

(*ii*) Rank of a matrix.

(*iii*) Row equivalent matrix.

(*iv*) Column rank of a matrix.

P.T.O.

(C) Reduce to row echelon form the matrix : 7

$$A = \begin{bmatrix} 1 & -2 & -1 & 4 \\ 2 & -4 & 3 & 5 \\ -1 & 2 & 6 & -7 \end{bmatrix}$$

Also find the row rank of A.

2. (A) (*i*) Prove that a system $Ax = B$ of n non-homogeneous equation in n unknowns has a unique solution provided A is non-singular *i.e.* $\rho(A) = n$. 8

(*ii*) For what values of λ, the equations : 7

$x + y + z = 1$

$x + 2y + 4z + \lambda$

$x + 4y + 10z = \lambda^2$

have a solution and solve completely in each case.

Or

(B) Express $\sin^n\theta$ in a series of cosines or sines of multiples of θ according as n is an even or odd integers. 8

(C) If $\sin\alpha + \sin\beta + \sin\gamma = \cos\alpha + \cos\beta + \cos\gamma = 0$, then show that : $\cos3\alpha + \cos3\beta + \cos3\gamma = 3\cos(\alpha + \beta + \gamma)$ and $\sin3\alpha + \sin3\beta + \sin3\gamma = 3\sin(\alpha + \beta + \gamma)$. 7

3. Attempt any *two* of the following : 10

(*i*) If :

$$A = \begin{bmatrix} 2+3i & i \\ 6i+5 & 0 \end{bmatrix}, B = \begin{bmatrix} i & 2i+1 \\ 2-i & -i \end{bmatrix},$$

then verify that :

$$\overline{AB} = \bar{A}\,\bar{B}$$

(*ii*) Find the rank of the matrix :

$$\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$$

using elementary matrix.

(*iii*) Check the following system of equations for consistency.

$$x - y + z + 1 = 0$$

$$x - y + z - 1 = 0$$

$$x - y - z + 1 = 0$$

(*iv*) Separate into its real and imaginary parts the expression $\tan(\alpha + \beta i)$.