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SB—91—2022

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (First Year) (First Semester) EXAMINATION

MAY/JUNE, 2022

(New Pattern)

MATHEMATICS

Paper-I

(Calculus)

(Thursday, 16-06-2022) **Time : 10.00 a.m. to 12.30 p.m.**

Time— 2.30 Hours *Maximum Marks*—40

N.B. :— (*i*) Attempt *all* questions.

(*ii*) Illustrate year answer with suitably labelled diagrams, wherever necessary.

1. (*a*) State and prove Leibnitz's theorem 8

(*b*) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, prove that : 7

$$x^2y_{n+2} + (2n + 1)xy_{n+1} + 2n^2y_n = 0$$

Or

(*a*) State and prove Taylor's theorem. 8

(*b*) Show that in the case of the curve $\beta y^2 = (x + \alpha)^3$, the square of the sub-tangent varies as the sub-normal. 7

2. State and prove Cauchy's mean value theorem and use Cauchy's mean value theorem to evaluate : 15

$$\lim_{x\to 1}\left[\frac{\cos\frac{1}{2}\pi x}{\log\left(\frac{1}{x}\right)}\right]$$

P.T.O.

Or

(*a*) If $z = \delta(x, y)$ is a homogeneous function of x, y of degree n, then

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z$$ 8

(*b*) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, $x \neq y$, show that : 7

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$$

3. Attempt any *two* of the following : 10

(*i*) Find the derivative of $\text{sech}^{-1}x$.

(*ii*) Expand $\log_e \cos(x + h)$ in powers of h by Taylor's theorem.

(*iii*) Prove that, for any quadratic function $px^2 + qx + r$, the value of θ in Lagrange's theorem is always 1/2 whatever p, q, r, a, h may be.

(*iv*) If $u = e^{xyz}$, show that :

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$$