SB—125—2022

FACULTY OF SCIENCE

B.Sc. (First Year) (Second Semester) EXAMINATION

MAY/JUNE, 2022

(New Course)

MATHEMATICS

Paper IV

(Analytical Solid Geometry)

(Monday, 20-6-2022)   **Time : 10.00 a.m. to 12.30 p.m.**

Time— 2.30 Hours   *Maximum Marks—40*

N.B. :—   (*i*)   *All* questions are compulsory.

         (*ii*)   Illustrate your answers with suitably labelled diagrams, wherever necessary.

1.   Attempt the following :   15

Prove that the angle between the lines whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2) is $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$. Also discuss the condition of parallelism and perpendicularity. Find the angle between the two planes $2x - y + 2z$ and $3x + 6y + 2z = 4$.

Or

(*a*)   Prove that the perpendicular distance of the point $P(x_1, y_1, z_1)$ from the plane $lx + my + nz = p$ is $lx_1 + my_1 + nz_1 - p$. Also find the length of the perpendicular.   8

P.T.O.

(b) The vertices of a triangle ABC are the points $(-1, 2, -3)$, $(5, 0, -6)$ and $(0, 4, -1)$ respectively. Determine the direction ratios of the bisector of the angle BAC. 7

2. Attempt the following : 15

What is the general equation of a sphere ? Find the equation to a sphere on line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) as a diameter. Find the equation of the line through the point $(1, 2, 3)$ and parallel to the line $x - y + 2z = 5$ and $3x + y + z = 6$.

Or

(a) Find the condition that two given straight lines : 8

$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}, \quad \frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$$

are coplanar. Also show that the lines $\dfrac{x + 3}{2} = \dfrac{y + 5}{3} = \dfrac{z - 7}{-3}$,

$= \dfrac{x + 1}{4} = \dfrac{y + 1}{5} = \dfrac{z + 1}{-1}$ are coplanar.

(b) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point $(1, 2, 3)$. 7

3. Attempt any *two* of the following : 10

(a) Find the equation to the right circular cone whose vertex is $P(2, -3, 5)$, axis PQ which makes equal angles with the axes and semi-vertical angle is $30°$.

(b) Find the magnitude and the equations of the line of shortest distance between the lines :

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}, \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}.$$

(c) Find the distance between the parallel planes $x + 2y - 3z + 1 = 0$ and $2x + 4y - 4z + 5 = 0$.

(d) Show that the direction cosines of a line joining the two points (x_1, y_1, z_1), and (x_2, y_2, z_2) are proportional to $x_2 - x_1, y_2 - y_1, z_2 - z_1$.