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SB—99—2022

FACULTY OF SCIENCE

B.Sc. (Second Year) (Third Semester) EXAMINATION

MAY/JUNE, 2022

(New Course)

MATHEMATICS

Paper VI

(Real Analysis-I)

(Thursday, 16-06-2022)   **Time : 2.00 p.m. to 4.30 p.m.**

Time— 2½ Hours   *Maximum Marks—40*

N.B. :— (i) *All* questions are compulsory.

    (ii) Figures to the right indicate full marks.

1. (A) If S and T are subsets of real numbers, then show that :   15

    (i) $S \subseteq T \Rightarrow S' \subseteq T'$ and

    (ii) $(S \cup T)' \; S' \cup T'$

<div align="center">Or</div>

  (A) Prove that Every convergent sequence is bounded.   8

  (B) Show that :   7

$$\lim_{n \to \infty} \frac{3 + 2\sqrt{n}}{\sqrt{n}} = 2$$

2. (A) If the alternating series $u_1 - u_2 + u_3 - u_4 + \ldots\ldots (u_n > 0, \; \forall \; n)$ is such that :   15

    (i) $u_{n+1} \leq u_n, \; \forall \; n$ and

    (ii) $\lim_{n \to \infty} u_n = 0$.

Then show that the series converges.

<div align="right">P.T.O.</div>

Or

(A) If Σu_n is a positive term series, such that $\underset{n \to \infty}{\lim} (u_n)^{1/n} = l$, then prove that the series : 8

(*i*) converges, if $l < 1$

(*ii*) diverges, if $l > 1$ and

(*iii*) the test fails to give any definite information, if $l = 1$.

(B) Show that the series :

$$\frac{1.2}{3^2.4^2} + \frac{3.4}{5^2.6^2} + \frac{5.6}{7^2.8^2} + \cdots\cdots \text{is convergent.}$$ 7

3. Attempt any *two* of the four : 5 marks each

(A) Prove that the union of an arbitrary family of open sets is open.

(B) Prove that a sequence cannot converge to more than one limit point.

(C) Show that the series :

$$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots\cdots \text{converges.}$$

(D) Test the convergence of the series whose n^{th} term is $\{(nq + 1)^{1/3} - n\}$.