This question paper contains **3** printed pages]

SB—114—2022

FACULTY OF SCIENCE

B.Sc. (Second Year) (Fourth Semester) EXAMINATION

MAY/JUNE, 2022

(Old Course)

MATHEMATICS

Paper IX

(Real Analysis-II)

(Friday, 17-06-2022) **Time : 2.00 p.m. to 4.30 p.m.**

Time— 2½ *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Figures to the right indicate full marks.

1. Prove that : 15

(*i*) If a bounded function f is integrable on $[a, b]$, then it is also integrable on $[a, c]$ and $[c, b]$ where c is a point of $[a, b]$.

(*ii*) Conversely if f is bounded and integrable on $[a, c]$, $[c, b]$ then it is also integrable on $[a, b]$.

(*iii*) In either case :

$$\int_a^b f\,dx = \int_a^c f\,dx + \int_c^b f\,dx, a \le c \le b.$$

Or

(*a*) Show tha if f is bounded function on $[a, b]$, then to every $\in > 0$ there corresponds $\delta > 0$ such that :

P.T.O.

(*i*) $U(p, f) < \overline{\int_a^b} f\, dx + \in$

(*ii*) $L(p, f) > \underline{\int_a^b} f\, dx - \in$

for every partition P of $[a, b]$ with norm $\mu(p) > \in$.

(*b*) Prove that; A necessary and sufficient condition for the integrability of a bounded function f is that to every $\in > 0$, ther *e* corresponds $\delta > o$ such that for every partition p of $[a, b]$ with norm $\mu(p) < \delta$, $U(p, f) - L(p, f) < \in$. 7

2. Prove that if a function f is bounded and integrable on $[a, b]$, then the function F defined as $F(x) = \int_a^x f(t)\, dt$, $a \le x \le b$ is continuous on $[a, b]$ and furthermore, if f is continuous at a point c of $[a, b]$ then, F is derivable at c and $F'(c) = f(c)$.

Or

(*a*) If f and g be two positive functions such that $f(x) \le g(x)$ for all x in $[a, b]$ prove that : 8

(*i*) $\int_a^b f\, dx$ converges, if $\int_a^b g dx$ converges and

(*ii*) $\int_a^b g dx$ diverges if $\int_a^b f dx$ diverges.

(*b*) Prove that improper integral $\int_a^b f dx$ converges at a if and only if to every $\in > 0$ there corresponds $\delta > 0$ such that : 7

$$\left|\int_{a+\lambda_1}^{a+\lambda_2} f dx\right| < \in,\ 0 < \lambda_1, \lambda_2 < \delta$$

3. Attempt any *two* of the following : 10

(*a*) For a periodic function of period 2π.

prove that :

(*i*) $\int_{\alpha}^{\beta} f dx = \int_{\alpha+2\pi}^{\beta+2\pi} f dx$

(*ii*) $\int_{-\pi}^{\pi} f dx = \int_{\alpha}^{\alpha+2\Pi} f dx$

where α, β being any numbers.

(*b*) Show that for a bounded integral function ϕ :

$$\lim \int_0^a \phi \frac{\sin nx}{\sin x} dx = \lim \int_0^a \phi \frac{\sin nx}{x} dx$$

$$0 < a < \pi$$

(*c*) If f is a periodic function with period 2Π defined as :

$$f(x) = \begin{cases} -1 \text{ for } -\pi < x < 0 \\ 1 \text{ for } 0 \le x \le \pi \end{cases}.$$

Expland in a series of sines and cosines of multiple angles of x.

(*d*) Find the Fourier series generated by the periodic function $|x|$ of period 2π.