This question paper contains **2** printed pages]

SB—133—2022

FACULTY OF SCIENCE

B.Sc. (Second Year) (Fourth Semester) EXAMINATION

MAY/JUNE, 2022

(New Course)

MATHEMATICS

Paper-X

(Ring Theory)

(Monday, 20-06-2022) **Time : 2.00 p.m. to 4.30 p.m.**

Time— 2½ *Hours* *Maximum Marks*—40

N.B. :—Figures to the right indicate full marks.

1. Define integral domain. If R is a ring, then prove that for all $a, b, c \in$ R. 15

   (*i*) $a(-b) = -(ab) = (-a)b$

   (*ii*) $a(b - c) = ab - ac$.

   Also if R is a ring such that $a^2 = a \ \forall \ a \in$ R, prove that $a + a = 0 \ \forall \ a \in$ R and $a + b = 0 \Rightarrow a = b$.

Or

   (*a*) If S is an ideal of a ring, R prove that the set $R|S = \{s + a : a \in R\}$ of all residue classes of sin R forms a ring for two compositions in R|S defined by for $s + a$, $s + b \in R|S$ 8

$$(s + a) + (s + b) = s + (a + b)$$

$$(s + a).(s + b) = s + ab$$

   (*b*) Prove that the homomorphism ϕ of a ring R into a ring R' is an isomorphism of R into R' if and only if $I(\phi) = (0)$, where $I(\phi)$ denotes the kernel of ϕ. 7

P.T.O.

2. Prove that A commutative ring with zero divisors can be imbedded in a field. Also for any given element of a ring R. Let 15

$$Ra = \{xa : x \in R\}$$

Then prove that Ra is a left ideal of R.

Or

(*i*) Prove that the set R[x] of all polynomials over an arbitrary ring R is a ring with respect to addition and multiplication of polynomials. 8

(*ii*) Solve : Add and multiply polynomials

$$f(x) = 2x^0 + 5x + 3x^2,\ g(x) = 1x^0 + 4x + 2x^3$$

over the Ring $(I_6, +_6, X_6)$. 7

3. Attempt any *two* of the following : 10

(*i*) If $f(x)$ and $g(x)$ be two non-zero polynomials over an arbitrary ring R, then deg $[f(x) + g(x)] \leq$ Max {deg $f(x)$, deg $g(x)$}.

(*ii*) Show that a field has no proper homomorphic image.

(*iii*) Show that the ring M of all 2 × 2 matrices with their elements as integers, the addition and multiplication of matrices being the two composition is a ring with zero divisors.

(*iv*) If R is a commutative ring, then prove that :

(i) $a | b$ and $b | c \Rightarrow a | c$

(ii) $a | b$ and $a | c \Rightarrow a | (b + c)$.