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SB—134—2022

FACULTY OF SCIENCE

B.Sc. (Second Year) (Fourth Semester) EXAMINATION

MAY/JUNE, 2022

(Old Pattern)

MATHEMATICS

Paper-X (SH-15-2021)

(Ring Theory)

(Monday, 20-6-2022) **Time : 2.00 p.m. to 4.30 p.m.**

Time— 2½ *Hours* *Maximum Marks*—40

N.B. :—Figures to the right indicate full marks.

1. If R is a ring, then for all $a, b \in R$ prove that : 15

(*i*) $a . 0 = 0 . a = 0$

(*ii*) $a(-b) = (-a)b = -(ab)$

(*iii*) $(-a)(-b) = ab$

(*iv*) $(-1)a = -a$ where 1 is unit in R

(*v*) $(-1)(-1) = 1$ where R has unit element 1.

Or

(*a*) If $R = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}\}$ is the set of integers mod 7 under the addition and multiplication mod 7, then find :

(*i*) $\overline{2} + \overline{3}$;

(*ii*) $\overline{4} + \overline{5}$;

(*iii*) $\overline{0} + \overline{6}$;

(*iv*) $\overline{2} . \overline{5}$;

(*v*) $\overline{4} . \overline{3}$. 8

(*b*) Prove that a Euclidean ring possesses a unit element. 7

P.T.O.

2. Define greatest common divisor and let R be a Euclidean Ring. Then any two elements a and b in R have greatest common divisor d. Moreover $d = \lambda a + \mu b$ for some $\lambda, \mu \in$ R. 15

Or

(*a*) If $f(x), g(x)$ be two non-zero elements of F[x], then prove that :

$$\deg [f(x) \,.\, g(x)] = \deg f(x) + \deg g(x).$$ 8

(*b*) Prove that if R is an integral domain so is R[x]. 7

3. Attempt any *two* of the following : 10

(*a*) If ϕ is a homomorphism of R into R′, then prove that :

(*i*) $\phi(0) = 0$

(*ii*) $\phi(-a) = -\phi(a)$ for every $a \in$ R.

(*b*) Let R be a Euclidean Ring. Suppose that for $a, b, c \in$ R, $a \mid bc$ but $(a, b) = 1$ then prove that $a \mid c$.

(*c*) If p is prime number of form $4n + 1$, then $p = a^2 + b^2$ for some integer a, b.

(*d*) Prove that $x^2 + 1$ is irreducible over integer mod 7.