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SB—113—2022

FACULTY OF SCIENCE

B.Sc. (Second Year) (Fourth Semester) EXAMINATION

MAY/JUNE, 2022

(New Pattern)

MATHEMATICS

Paper IX

(Real Analysis-II)

(Friday, 17-06-2022) **Time : 2.00 p.m. to 4.30 p.m.**

Time— 2½ *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *all* questions.

(*ii*) Figures to the right indicate full marks.

1. If p^* is a refinement of a partition p and $|f(x)| \leq k$, for all $x \in [a, b]$, then show that : 15

(*i*) $L(p^*, f) \geq L(p, f)$ and

(*ii*) $L(p^*, f) \leq U(p, f)$ and

(*iii*) $L(p, f) \leq L(p^*, f) \leq L(p, f) + 2pk\mu$

Or

(*a*) A function f is integrable over $[a, b]$ iff for $\in > 0 \; \exists \; \delta > 0$ if p, p' are any two partitions of $[a, b]$ with mesh less than δ, then prove that : 8

$$|S(p, f) - S(p', f)| < \in$$

(*b*) If a function f is continuous on $[a, b]$, then prove that there exists a number ξ in $[a, b]$ such that : 7

$$\int_a^b f\, dx = f(\xi)(b-a)$$

P.T.O.

2. If a function f is bounded and integrable on $[a, b]$, then prove that the function F defined as $F(n) = \int_a^b f(t)dt,\ a \le x \le b$ is continuous on $[a, b]$, and further more, show that if f is continuous at a point C on $[a, b]$ then F is derivable at C and $F'(c) = f(c)$.

Also show that : 10+5=15

$$\int_0^t f dx = 1 - \cos t$$

where $f(x) = \sin x$.

3. Attempt any *two* :

(*a*) If f is bounded and integrable on $[a, b]$, then prove that $|f|$ is also bounded and integrable on $[a, b]$. 5

(*b*) If a function f is monotonic on $[a, b]$, then prove that it is integrable on $[a, b]$. 5

(*c*) Show that $\int_0^1 \frac{\sin 1/x}{x^p} dx$, $p > 0$ converges absolutely for $p > 1$. 5

(*d*) If f and g are positive and $f(x) \le g(x)$, for all x in $[a, X]$, then prove that : 5

(*i*) $\int_a^\infty f dx$ converges, if $\int_a^\infty g dx$ converges.

(*ii*) $\int_a^\infty g dx$ diverges, if $\int_a^\infty f dx$ diverges.