This question paper contains **2** printed pages]

SB—44—2022

FACULTY OF SCIENCE

B.Sc. (Sixth Semester) EXAMINATION

MAY/JUNE, 2022

(CBCS/New Pattern)

MATHEMATICS

Paper-XV

(Complex Analysis)

(Saturday, 11-06-2022) **Time : 10.00 a.m. to 12.30 p.m.**

Time— 2.30 Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Explain the method to find *n*th roots of non-zero complex number z_0. Also find the cube root of $(-8i)$. 15

Or

(*a*) Derive Cauchy–Riemann equation in cartesian form. 8

(*b*) Let $f(z) = u(x, y) + iv(x, y)$. If $f(z)$ and $\overline{f(z)}$ are both analytic in given domain D, then show that $f(z)$ is constant throughout D. 7

2. Let *f* be analytic function everywhere inside and on a simple closed contour C, taken in the positive sense. If z_0 is any interior point to C, then prove that : 15

$$f(z_0) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z - z_0}\, dz$$

P.T.O.

Hence, evaluate $\int_C \frac{z\,dz}{(9-z^2)(z+i)}$ where C is positively oriented circle $|z| = 2$.

Or

(*a*) If $w(t)$ is a piecewise continuous complex valued function defined on an interval $a \le t \le b$, then prove that : 8

$$\left|\int_a^b w(t)dt\right| \le \int_a^b |w(t)|\,dt$$

(*b*) Let $w(t)$ be continuous complex valued function of t defined on an interval $a \le t \le b$, then prove that it is not necessarily true that there is a number c in interval $a < t < b$ such that $\int_a^b w(t)dt = w(c)\,(b - a)$.

Also evaluate $\int_0^1 (1+it)^2\,dt$. 7

3. Attempt (any *two*) of the following : 5 each

(*i*) Show that a set S is open if and only if each point in S is an interior point.

(*ii*) If $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then prove that its component functions u and v are harmonic in D.

(*iii*) Find the value of $\int_c \bar{z}\,dz$ where c is the right hand half * of the circle $|z| = 2$ from $z = 2i$ & $z = -2i$

$$^*\left[z = 2e^{i\theta}, \left(-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right)\right]$$

(*iv*) If a function f is entire and bounded in the complex plane, then prove that f is constant throughout plane.