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SB—84—2022

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Fifth Semester) EXAMINATION

JUNE/JULY, 2022

(CBCS/Old Pattern)

MATHEMATICS

Paper XII

(Metric Spaces)

(Wednesday, 15-6-2022)

Time : 10.00 a.m. to 12.30 p.m.

Time—2½ Hours

Maximum Marks—40

N.B. :— (i) Attempt All questions.

(ii) Figures to the right indicate full marks.

1. Let (X, d) be any metric space, then prove that a subset F of X is closed if and only if its complement in X is open. Further prove that every closed sphere is a closed set. 15

Or

(a) Let A and B be any two sets of a metric space (X, d) , then prove that : 8

(i) \bar{A} is a closed set

(ii) If $A \subseteq B$, then $\bar{A} \subseteq \bar{B}$.

(b) Let (X, d) be any metric space, show that the function d_1 defined by : 7

$$d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}$$

for all $x, y \in X$, is a metric on X .

2. Define continuity in a metric space and prove that, if (X, d_1) and (Y, d_2) are two metric spaces, then $f : X \rightarrow Y$ is continuous if and only if $f^{-1}(G)$ is open in X , whenever G is open in Y . 15

P.T.O.

Or

- (a) Define Cauchy sequence and prove that every convergent sequence is a Cauchy sequence. 8
- (b) Let (X, d_1) and (Y, d_2) be metric spaces, show that $f: X \rightarrow Y$ is continuous if and only if $f(\overline{A}) \subseteq \overline{f(A)}$, for every $A \subseteq X$. 7
3. Attempt any *two* of the following : 5 each
- (a) Prove that every compact subset of A of a metric space (X, d) is bounded.
- (b) Let X be an infinite set with the discrete metric space, then show that (X, d) is not compact.
- (c) Prove that if Y is a connected subset of a metric space (X, d) , then Y cannot be expressed as disjoint union of two non-empty closed sets in Y .
- (d) Show that $D = \left\{ (x, y) : x \neq 0; y = \sin \frac{1}{x} \right\}$ is disconnected subset of \mathbb{R}^2 .