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# SB—103—2022

## FACULTY OF SCIENCE AND TECHNOLOGY

**B.Sc. (Third Year) (Fifth Semester)**

**EXAMINATION**

**MAY/JUNE, 2022**

**(New/CBCS Course)**

MATHEMATICS

Paper XIII

(Linear Algebra)

**(Friday, 17-06-2022)** **Time : 10.00 a.m. to 12.30 p.m.**

*Time*— *2.30 Hours* *Maximum Marks*—40

*N.B.* :— (*i*) Attempt *all* questions.

(*ii*) Illustrate your answers with suitably labelled diagrams, wherever necessary.

1. Prove that a non-empty subset S of a vector space V is a subspace of V if and only if the following conditions are satisfied : 15

(*i*) If $u$ , $v \in$ S then U + V $\in$ S.

(*ii*) If $u \in$S and $\alpha$ is a scalar then $\alpha\, u \in$ S and using above property. Show that the set $S = \{p \in \rho / p(x_0) = 0\}$ of all polynomials $p \in$ P which vanishes at a fixed point $x_0$ is a subspace of P.

*Or*

Answer each of the following :

(*a*) If T : U $\rightarrow$ V be a linear map, then prove that T is one-one iff N(T) is zero subspace, $\{O_U\}$ of U. 8

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(*b*) Prove that the set {(1, 1, 1) (1 –1, 1), (0, 1, 1) is a basis for a vector space $V_3$. 7

2. For vectors *u* and $v \neq 0$ in an inner product space V, define vector projection of *u* along *v* and prove that every finite-dimensional inner product space has an orthonormal basis. 15

*Or*

Answer each of the following :

(*a*) Let $T : U \to V$ be a non-singular linear map. Then prove that $T^{-1} : V \to U$ is linear, one-one and onto map. 8

(*b*) If $S : U \to V$ is a linear map, $\alpha$ is a scalar and $P : U \to V$ is defined by $p(u) = \alpha\ [S(u)]$ for all $u \in U$, then prove that *p* is linear. 7

3. Attempt any *two* of the following : 10

(*a*) Show that the vectors (1, 0, 1), (1, 1, 0) and (1, 1, –1) in $V_3$ are linearly independent.

(*b*) Prove that if a vector space V has a basis of *n*-elements, then every other basis of V also has *n*-elements.

(*c*) Let $T : U \to V$ and $S : V \to W$ are two linear maps. If S and T are non-singular, then prove that ST is non-singular and $(ST)^{-1} = T^{-1}S^{-1}$.

(*d*) Determine all the eigenvalues of :

$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$



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