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SB—104—2022

FACULTY OF SCIENCE

B.Sc. (Third Year) (Fifth Semester) EXAMINATION

JUNE/JULY, 2022

(CBCS/Old Pattern)

MATHEMATICS

Paper XIII

(Linear Algebra)

(Friday, 17-6-2022) **Time : 10.00 a.m. to 12.30 p.m.**

Time—2½ *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Illustrate your answers with suitably labelled diagrams wherever necessary.

(*iii*) Figures to the right indicate full marks.

1. If V and W are of dimensions 'm' and 'n' respectively over F, then prove that Hom (V, W) is of dimension 'mn' over F.

Also if 'F' is the field of real numbers and W is a subspace of V = R^3, spanned by (1, 2, 3) and (0, 4, –1) then, find A(W). 15

Or

(*a*) If $v_1, v_2, v_3, ..., v_n$ in V have W as linear span and if $v_1, v_2, ..., v_k$ are linearly independent, then we can find a subset of $v_1, v_2 ...., v_n$ of the form $v_1, v_2, ..., v_k, \; v_{i_1}, v_{i_2} ....., \; v_{i_r}$ consisting of linearly independent elements whose linear span is also W. 8

(*b*) Find the rank of the following system of homogeneous linear equations over F, the field of real numbers and find all the solutions of : 7

$$x_1 + 2x_2 - 3x_3 + 4x_4 = 0$$

$$x_1 + 3x_2 - x_3 = 0$$

$$6x_1 + x_3 + 2x_4 = 0$$

P.T.O.

2. Let V be a finite-dimensional inner product space then pove that V has an orthonormal set as a basis. Also if $\{w_1, w_2, ..., w_m\}$ is an orthonormal set in V, prove that : 15

$$\sum_{i=1}^{m} |(w_i, v)|^2 \leq \| v \|^2$$

for any $v \in$ V.

Or

(*a*) If V is a finite dimensional inner product space and if W is a subspace of V, then prove that $V = W + W^{\perp}$. More particularly V is the direct sum of W and $W^{\perp}$. 8

(*b*) Let V be the real functions $y = f(x)$ satisfying $\frac{d^2y}{dx^2} + 9y = 0$, then prove that V is a two-dimensional real vector space. 7

3. Attempt any *two* of the following : 10

(*a*) If $\lambda \in$ F is a characteristic root of T $\in$ A(V), then for any polynomial $q(x) \in$ F[x], $q(\lambda)$ is a characteristic root of q(T).

(*b*) Prove that the regular elements in A(V) form group.

(*c*) Let A be an algebra with unit element over F and suppose that A is of dimension m over F, then prove that every element in 'A' satisfies some non-trivial polynomial in F[x] of degree at most 'm'.

(*d*) Compute the following matrix products :

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & -1 & 2 \\ 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ -1 & -1 & -1 \end{pmatrix}.$$