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SB—132—2022

FACULTY OF SCIENCE

B.Sc. (Third Year) (Fifth Semester) EXAMINATION

JUNE/JULY, 2022

(CBCS/Old Pattern)

MATHEMATICS

Paper XIV

(Complex Analysis)

(Monday, 20-6-2022)   **Time : 10.00 a.m. to 12.30 p.m.**

Time—2½ Hours   *Maximum Marks—40*

N.B. :— (i)   *All* questions are compulsory.

(ii)   Figures to the right indicate full marks.

1.   Discuss the method to find nth root of non-zero complex number 'z_0'. Hence
     determine the nth root of unity.   15

Or

(a)   Use de-Moivre's formula to derive the following trigonometric
      identities :   8

(i) $\cos 3\theta = \cos^3 \theta - 3 \cos \theta \cdot \sin^2 \theta$

(ii) $\sin 3\theta = 3 \cos^2 \theta \cdot \sin \theta - \sin^3 \theta.$

(b)   Prove that :   7

(i) z is real if and only if $\bar{z} = z$.

(ii) z is either real or pure imaginary if and only if $\bar{z}^2 = z^2$

        where $\bar{z}$ – conjugate of $z = x + iy$.

2.   If a function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D. Then
     prove that its component function u and v are harmonic in D. Moreover show
     that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate
     $v(x, y)$.   15

P.T.O.

Or

(*a*) Suppose that : 8

$$f(z) = u(x, y) + iv(x, y),$$

$$z_0 = x_0 + iy_0 \text{ and } w_0 = u_0 + iv_0$$

Then prove,

$$\lim_{z \to z_0} f(z) = w_0$$

if and only if

$$\lim_{(x, y) \to (x_0, y_0)} u(x, y) = u_0 \text{ and}$$

$$\lim_{(x, y) \to (x_0, y_0)} v(x, y) = v_0$$

(*b*) Suppose that a function $f(z) = u(x, y) + iv(x, y)$ and its conjugate

$$\overline{f(z)} = u(x, y) - iv(x, y)$$

are both analytic in a given domain D. Then show that $f(z)$ is constant throughout D. 7

3. Attempt any *two* of the following : 10

(*a*) Find the principal value of :

(*i*) $(- i)^i$

(*iii*) $z^{2/3}$.

(*b*) Explain Logarithmic function for non-zero complex variable '*z*'.

(*c*) Find all the values of z such that $e^z = - 2$.

(*d*) Find the derivatives :

(*i*) $\dfrac{d}{dz} (\sinh z)$

(*ii*) $\dfrac{d}{dz} (\cosh z)$.