SB—83—2022

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Fifth Semester) EXAMINATION

MAY/JUNE, 2022

(CBCS/New Pattern)

MATHEMATICS

Paper-XII

(Metric Spaces)

(Wednesday, 15-06-2022)   **Time : 10.00 a.m. to 12.30 p.m.**

Time— 2.30 Hours   *Maximum Marks—40*

N.B. :—   (i)   Attempt *All* questions.

       (ii)   Figures to the right indicate full marks.

1.   In any metric space (X, d), prove that :   15

     (i)   The union of an arbitrary family of open sets is open.

     (ii)   The intersection of a finite number of open sets is open

Or

     (a)   Let (X, d_1) and (Y, d_2) be two metric spaces, then prove that $f : X \to Y$ is continuous if and only if $f^{-1}(G)$ is open in X, whenever G is open in Y.   8

     (b)   Let (X, d_1) and (Y, d_2) be metric spaces. Show that $f : X \to Y$ is continuous if and only if $f(\overline{A}) \subseteq \overline{f(A)}$, for every $A \subseteq X$.   7

2.   Prove that every compact subset F of a metric space (X, d) is closed : 15

Or

     (a)   Prove that continuous image of a connected set is connected.   8

     (b)   Discuss the connectedness of the following subset of the Euclidean space R^2 :   7

       $D = \{(x, y) : x \neq 0 \text{ and } y = \sin 1/x\}$

P.T.O.

3. Attempt any *two* of the following : 5 each

    (*i*)   Prove that every convergent sequence is a Cauchy sequence.

    (*ii*)   Prove that every compact subset A of a metric space (X, *d*) is bounded.

    (*iii*)   Prove that every open sphere is a neighbourhood of each of its points.

    (*iv*)   Prove that the union of two connected sets, having non-empty inter-section is connected.