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**SB—69—2022**

**FACULTY OF SCIENCE**

**B.Sc. (Third Year) (Sixth Semester) EXAMINATION**

**MAY/JUNE, 2022**

**(CBCS/New Pattern)**

**MATHEMATICS**

**Paper-XVI**

**(Integral Transforms)**

**(Tuesday, 14-06-2022)**

**Time : 10.00 a.m. to 12.30 p.m.**

*Time— 2½ Hours*

*Maximum Marks—40*

*N.B. :— (i) All questions are compulsory.*

*(ii) Figures to the right indicate full marks.*

1. If  $L[f(t)] = F(s)$ , then prove that  $L\left[\frac{1}{t}f(t)\right] = \int_s^\infty F(s)ds$ . 15

Hence, find the Laplace transform of  $\frac{\sin 2t}{t}$

*Or*

(a) Find the Inverse Laplace transform of  $\frac{S+4}{S(S-1)(S^2+4)}$  8

(b) Find the Inverse Laplace transform of  $\frac{S^2+3}{S(S^2+9)}$  7

2. Derive the Fourier sine integral  $f(x) = \frac{2}{\pi} \int_0^\infty \sin ux du \int_0^\infty f(t) \sin ut dt$  and the

Fourier cosine integral  $f(x) = \frac{2}{\pi} \int_0^\infty \cos ux du \int_0^\infty f(t) \cos ut dt$ . 15

P.T.O.

Or

- (a) Applying the convolution, solve the following initial value problem : 8

$$y'' + y = \sin 3t$$

$$y(0) = 0, y'(0) = 0$$

- (b) Solve :  $\frac{dx}{dt} + y = 0$  and  $\frac{dy}{dt} - x = 0$  7

under the condition  $x(0) = 1, y(0) = 0$ .

3. Attempt any *two* of the following : 5 marks each

- (i) Find the Laplace transform of  $\cos^2 t$ .

- (ii) Find the Inverse Laplace transform of  $\frac{S}{4S^2 - 25}$ .

- (iii) Using the Laplace transforms, find the solution of the initial value problem :

$$y'' + 25y = 10 \cos 5t$$

$$y(0) = 2, y'(0) = 0$$

- (iv) Find the Fourier transform of :

$$f(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$$