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SB—70—2022

FACULTY OF SCIENCE

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

JUNE/JULY, 2022

(CBCS/Old Course)

MATHEMATICS

Paper XVI

(Integral Transforms)

(Tuesday, 14-6-2022)

Time : 10.00 a.m. to 12.30 p.m.

Time—2½ Hours

Maximum Marks—40

N.B. :- (i) All questions are compulsory.

(ii) Figures to the right indicate full marks.

1. Answer any of the following : 15

If $L[f(t)] = F(s)$, then prove that $L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} [F(s)]$. Also obtain the Laplace transform of $t^2 e^t \cdot \sin 4t$.

Or

(a) If $L[f_1(t)] = F_1(s)$ and $L[f_2(t)] = F_2(s)$, then prove that : 8

$$L\left\{\int_0^t f_1(x) \cdot f_2(t-x) dx\right\} = F_1(s) \cdot F_2(s)$$

(b) Find the Laplace transform of $t^2 \cos at$. 7

2. Answer any of the following : 15

Find the inverse Laplace transform of $\frac{s^2 + 3}{s(s^2 + 9)}$.

Also solve the initial value problem

$$2y'' + 5y' + 2y = e^{-2t}, y(0) = 1, y'(0) = 1$$

using the Laplace transforms.

P.T.O.

Or

(a) Find the value of :

7

$$\mathcal{L}^{-1} \left\{ \frac{1}{(s^2 + a^2)^2} \right\}.$$

(b) Find the inverse Laplace transform of :

8

$$\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}.$$

3. Attempt any *two* of the following :

5 each

(a) If $F(s)$ is the complex Fourier transform of $f(x)$, then prove that :

$$\mathcal{F}\{f(x - a)\} = e^{isa} F(s).$$

(b) Express the function :

$$f(x) = \begin{cases} 1 & \text{when } |x| \leq 1 \\ 0 & \text{when } |x| > 1 \end{cases}$$

as a Fourier integral. Hence evaluate :

$$\int_0^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda.$$

(c) Find the Fourier sine and cosine transform of $f(x) = e^{-ax}$.(d) If $\mathcal{F}[f(x)] = F(s)$, then prove that :

$$\mathcal{F}\{x^n f(x)\} = (-i)^n \frac{d^n}{ds^n} F(s).$$