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# SB—93—2022

## FACULTY OF SCIENCE & TECHNOLOGY

## B.Sc. (Third Year) (Sixth Semester) EXAMINATION

## JUNE/JULY, 2022

## (CBCS/New Pattern)

MATHEMATICS

Paper – XVII

(Topology)

**(Thursday, 16-6-2022)** **Time : 10.00 a.m. to 12.30 p.m.**

*Time*—2½ *Hours* *Maximum Marks*—40

*N.B.* :— (*i*) Attempt either A or B for question No. **1** and **2**.

(*ii*) *All* symbols carry usual meanings.

(*iii*) Figures to the right indicate full marks.

1. (A) Attempt the following :

(*a*) Let X be a topological space. Suppose that C is a collection of open sets of X such that for each open set U of X and each $x$ in U, there is an element C of C such that $x \in C \subset U$. Then show that C is a basis for the topology on X. 8

(*b*) Show that topologies of $R_l$ and $R_k$ are strictly finner than the standard topology on R. 7

*Or*

(B) Attempt the following : 15

(*i*) If A is a subspace of X and B is a subspace of Y, then show that the product topology on A × B is same as the topology A × B inherits as a subspace of X × Y.

(*ii*) Let X be an ordered set in the order topology : Let Y be a subset of X that is convex in X. Then show that the order topology on Y is the same as the topology Y inherits as a subspace of X.

P.T.O.

2. (A) Let X and Y be topological spaces. Let $f : X \to Y$. Then show that the following are equivalent : 15

(*i*) $f$ is continuous.

(*ii*) For every subset A of X, one has $f(\overline{A}) = \overline{f(A)}$.

(*iii*) For every closed set B of Y, the set $f^{-1}$ (B) is closed in X

(*iv*) For each $x \in X$ and each neighborhood V of $f(x)$, there is a neighborhood U of $x$ such that $f$ (U) $\subset$ V.

*Or*

(B) Attempt the following :

(*a*) Let Y be a subspace of X. Then show that set A is closed in Y if and only if it equals the intersection of a closed set of X with Y. 8

(*b*) Define Hausdorff space. Hence show that subspace of a Hausdorff sapce is Hausdorff. 7

3. Attempt any *two* of the following : 10

(*i*) Find any *five* posssible topologies for a set X = {$a$, $b$, $c$}.

(*ii*) If $\beta$ is a basis for the topologies on X, then show that the collection $\beta_y = \{B \cap Y \mid B \in \beta\}$ is a basis for the subspace topology on Y.

(*iii*) Let $A \subset X$ and $B \subset Y$. Show that in the space X × Y, $\overline{A \times B} = \overline{A} \times \overline{B}$.

(*iv*) If the sets C and D form a separation of X, and if Y is connected subspace of X, then Y lies entirely within either C or D.