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SB—96—2022

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Sixth Semester) EXAMINATION

JUNE/JULY, 2022

(CBCS/Old Pattern)

MATHEMATICS

Paper-XVII

(Topology)

(Thursday, 16-6-2022) **Time : 10.00 a.m. to 12.30 p.m.**

Time—2½ Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Let B a non empty set, let n be a positive integer then prove that following are equivalent : 15

(*i*) There is a surjective function $f : \{1, 2, ..., n\} \to$ B.

(*ii*) There is an injective function g : B $\to \{1, 2, ..., n\}$

(*iii*) B is finite and has at most n elements.

Also prove that the number of elements in a finite set A is uniquely determined by A.

Or

(*a*) Define the product topology on X × Y. 8

If β is a basis for the topology on X, and C is a basis for the topology of Y, then prove that collection D = {B × C | B ∈ β and C ∈ C} is a basis for the topology of X × Y.

(*b*) Let Y be a subspace of X. Then prove that a set A is closed in Y if andonly if it equlas the intersection of a closed st of X with Y.

2. Let A be a subset of the topological spae X, then prove that : 15

(*i*) $x \in \bar{A}$ if and only if every open set U containing x intersects A.

(*ii*) Supposing the topology of X is a given by a basis, $x \in \bar{A}$ if and only if every basis element B containing x intersects A.

Also define the Hausdorff space and prove that every finite point set in a Hausdorff space X is closed.

P.T.O.

Or

(*a*) Let n be a positive integer, A be a set and a_0 be an element of A. Then prove that there exists a bijective correspondence f of the set A with the set {1, 2, ..., n + 1} if and only if there exists a bijective correspondence g of the set A – {a_0} with {1, 2, ..., n}. 8

(*b*) Define projection maps π_1, and π_2. Also prove that the collection : 7

$$S = \{\pi_1^{-1}\ U | U \text{ oen in } X\}\ U$$
$$\{\pi_2^{-1}(V) | V \text{ open in } Y\}$$

is a sub basis for the product topology on X × Y.

3. Attempt any *two* of the following : 5 each

(*a*) Prove that the lower limit topology T^1 R is strictly finer than the standard topology T.

(*b*) If A is a subspace of X and B is a subspace of Y, then prove that the product topology on A × B is the same as the topology A × B inherits as a subspace of X × Y.

(*c*) Let A be a subset of thetopological space X, let A'be the set of all limit points of A, then prove that $\bar{A} = A \cup A'$.

(*d*) Let X = {a, b, c, d} T_1 = {ϕ, X {b}, {b, c} and T_2 = {ϕ, X, {b}, {a, b}}. Find the smallest topology containing T_1 and T_2, also find the largest topology contained in T_1 and T_2.