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SB—98—2022

FACULTY OF SCIENCE

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

JUNE/JULY, 2022

(CBCS/Old Pattern)

MATHEMATICS

Paper XVII

(Elementary Number Theory)

(Thursday, 16-6-2022) **Time : 10.00 a.m. to 12.30 p.m.**

Time—2½ *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Prove that for given integers a and b, not both of which are zero; there exist integers x and y such that

$$gcd\ (a, b) = ax + by$$

Hence find integers x and y satisfying

$$gcd\ (24, 138) = 24x + 138y.$$ 15

Or

(*a*) Prove that for given integers a and b, with $b > 0$, there exist unique integers q and r satisfying

$$a = qb + r,\ 0 \leq r < b.$$ 8

(*b*) State well-ordering principle and prove that if a and b are only positive intetgers, then there exists a positive integer n, such that $na \geq b$. 7

2. Prove that every positive integer $n > 1$ can be expressed as a product of primes; this representation is unique, apart from the order in which the factors occur. 15

P.T.O.

Or

(*a*) Show that the number $\sqrt{2}$ is irrational. 8

(*b*) If all the $n > 2$ terms of the arithmetic progression

$$p,\ p + d,\ p + 2d,\ \ldots\ldots,\ p + (n - 1)d$$

are prime numbers, then prove that the common difference d is divisible by every prime $q < n$. 7

3. Attempt any *two* of the following : 10

(*a*) Let $p(x) = \sum_{k=0}^{m} c_k x^k$ be a polynomial function of x with integral coefficients c_k.

If $a \equiv b \pmod{n}$, then prove that

$$p(a) \equiv p(b) \pmod{n}.$$

(*b*) Prove that for arbitrary integers a and b, $a \equiv b \pmod{n}$ if and only if a and b leave the same non-negative remainder when divided by n.

(*c*) Find the remainder when 2^{50} is divided by 7.

(*d*) Find the solutions of the system of congruences :

$$3x + 4y \equiv 5 \pmod{13}$$

$$2x + 5y \equiv 7 \pmod{13}$$