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NA—41—2023

FACULTY OF SCIENCE

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(CBCS/New Pattern)

MATHEMATICS

Paper XVI

(Integral Transforms)

(Monday, 11-12-2023) **Time : 10.00 a.m. to 12.00 noon**

Time—Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. If $L[f(t)]$ denote the Laplace transform of $f(t)$, then prove that : 15

$$L[f^n(t)] = s^n L[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) \,\ldots\ldots\ldots - f^{n-1}(0).$$

Hence find the Laplace transform of $t^2 \cos at$.

Or

(*a*) Find the inverse Laplace transform of : 8

$$\frac{s-1}{s^2-6s+25}.$$

P.T.O.

(*b*) Find the inverse Laplace transform of : 7

$$\frac{s^2}{(s^2+a^2)(s^2+b^2)}.$$

2. Prove the Fourier sine and cosine integrals : 15

$$f(x) = \frac{2}{\pi}\int_0^\infty \sin ux \, du \int_0^\infty f(t).\sin ut \, dt$$

and

$$f(x) = \frac{2}{\pi}\int_0^\infty \cos ux \, du \int_0^\infty f(t) \cos ut \, dt.$$

Or

(*a*) Using the Laplace transforms, find the solution of the initial value problem : 8

$$y'' + 25y = 10 \cos 5t,$$

$$y(0) = 2, \; y'(0) = 0.$$

(*b*) Solve the initial value problem : 7

$$2y'' + 5y' + 2y = e^{-2t},$$

$$y(0) = 1, \; y'(0) = 1.$$

3. Attempt any *two* of the following : 5 marks each

(*a*) Find the Laplace transform of $\cos^2 t$.

(*b*) Find the inverse Laplace transform of :

$$\frac{s^2+3}{s(s^2+9)}.$$

(*c*) Applying convolution, solve the following initial value problem :

$$y'' + y = \sin 3t,$$

$$y(0) = 0,\ y'(0) = 0.$$

(*d*) If F(*s*) is the complex Fourier transform of *f*(*x*), then prove that :

$$F\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right).$$