This question paper contains **2** printed pages]

NA—50—2023

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Third Year) (Fifth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(CBCS/New Pattern)

MATHEMATICS

Paper–XII

(Metric Spaces)

(Wednesday, 12-12-2023) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *all* questions.

(*ii*) Figures to the right indicate full marks.

1. Prove that every compact subset F, of a metric space (X, d), is closed. 15

Or

(*a*) Let (X, d) be any metric space. Prove that a subset F, of X, is closed if and only if its complement in X is open. 8

(*b*) Prove that every open sphere is a neighbourhood of each of its points. 7

2. Let Y be a subset of a metric space (X, d), then prove that the following are equivalent :

(*i*) Y is connected

P.T.O.

(*ii*) Y cannot be expressed as disjoint union of two non-empty closed sets in Y. 15

Or

(*a*) Let (X, d_1) and (Y, d_2) be two metric spaces, then prove that $f : X \to Y$ is continuous if and only if $f^{-1}(G)$ is open in X, whenever G is open in Y. 8

(*b*) Prove that every convergent sequence is a Cauchy sequence. 7

3. Attempt any *two* of the following : 5 each

(*i*) Let A and B be any two subsets of a metric space (X, *d*), then prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

(*ii*) If $f(x) = x^2$, $0 \le x \le \frac{1}{3}$. Then prove that f is a contraction mapping on $\left[0, \frac{1}{3}\right]$ with the usual metric *d*.

(*iii*) Prove that every compact subset A of a metric space (X, *d*) is bounded.

(*iv*) Discuss the connectedness of the subset :

$$D = \left\{(x, y) \mid x \neq 0, y = \sin\frac{1}{x}\right\}$$

of the Euclidean space R^2.