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NA—56—2023

FACULTY OF SCIENCE & TECHNOLOGY

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(CBCS/New Pattern)

MATHEMATICS

Paper-XVII

(Topology)

(Wednesday, 13-12-2023) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt either A or B for question Nos. 1 & 2.

(*ii*) *All* symbols carry usual meanings.

(*iii*) Figures to the right indicate full marks.

1. (A) Attempt the following :

(*i*) Define topology on a Set X. Let X be a three element set $X=\{a,b,c,\}$ then find any six topologies on X. 8

(*ii*) Show that the topologies of R_l and R_k are strictly finer than the standard topology on R. 7

Or

(B) Attempt the following :

(*i*) Define subspace topology. Hence show that if β is a basis for the topology on X, then the collection $\beta_y = \{B \cap Y / B \in \beta\}$ is a basis for the subspace topology on Y. 7

P.T.O.

(*ii*) If A is a subspace of X and B is a subspace of Y, then show that the product topology on A × B is the same as the topology A × B inherits as a subspace of X × Y. 8

2. (A) Let X and Y be topological spaces. Let $f : X \to Y$. Then show that the following are equivalent : 15

(*i*) f is continuous

(*ii*) For every subset A of X one has $f(\overline{A}) = \overline{f(A)}$.

(*iii*) For every closed set B of Y the set $\overline{f}^{1}(B)$ is closed in X.

(*iv*) For each $x \in X$ and each neighborhood V of $f(x)$ there is a neighborhood U of x such that $f(U) \subset V$.

Or

(B) Attempt the following :

(*i*) Define limit point. Let A be a subset of the topological spece X, let A' be the set of all limit points of A. Then show that : $\overline{A} = A \cup A'$. 8

(*ii*) Show that the product of two Hausdorff spaces is Hausdorff space.7

3. Attempt any *two* of the following : 5+5

(*a*) Let x be a set, let B be a basis for a topology λ on X. Then show that λ equals the collection of all union of element of β.

(*b*) Show that the collection

$S = \{\pi_1^{-1}, (U) / U \text{ is open in } U\}$

$\{\pi_2^{-1}, (V) / V \text{ is open in } Y\}$

is a sub-basis for the product topology on X × Y.

(*c*) Let Y be a subspace of X. Then show that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.

(*d*) Show that the subspace X of R where $X = \{0\} U \{1 / n / n \in Z_+\}$ is compact.