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NA—58—2023

FACULTY OF SCIENCE AND TECHNOLOGY

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(CBCS/New Pattern)

MATHEMATICS

Paper–XVII

(Elementary Number Theory)

(Wednesday, 13-12-2023) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Figures to the right indicate full marks.

(*ii*) Attempt *all* questions.

1. Let a and b be integers, not both zero, then prove that a and b are relatively primes if and only if there exists integers x and y such that $1 = ax + by$.

   Also show that if $gcd\ (a, b) = d$, then $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$. 15

Or

(*a*) If p is a prime and $p \mid ab$, then prove that $p \mid a$ or $p \mid b$. Also show that the number $\sqrt{2}$ is irrational. 8

(*b*) Use the sieve of eratosthens to find all the primes not exceeding 100. 7

2. Prove that for arbitrary integers a and b, $a \equiv b \pmod{n}$ if and only if a and b leave the same non-negative remainder when divided by n. Show that 41 divides $2^{20} - 1$. 15

P.T.O.

Or

(*a*) Let $n_1, n_2, ..., n_r$ be positive integers such that gcd $(n_i, n_j) = 1$, for $i \neq j$.

Prove that the system of linear congruences

$$x \equiv a_1 (\text{mod } n_1)$$
$$x \equiv a_2 (\text{mod } n_2)$$
$$\vdots$$
$$x \equiv a_r (\text{mod } n_r)$$

has a simultaneous solution, which is unique modulo the integer $n_1 \cdot n_2 ..... n_r$. 8

(*b*) Let p be a prime and suppose that $p \not| a$, then prove that $a^{p-1} \equiv 1 (\text{mod } p)$. 7

3. Attempt any *two* of the following : 10

(*a*) If n is an odd pseudoprime, then prove that $M_n = 2^n - 1$ is a larger one.

(*b*) Show that the integers 1,571,724 is divisible by 9 and 11.

(*c*) Find the canonical form of the integers 4725 and 17460.

(*d*) By using Euclidean algorithm find *gcd* (12378, 3054).