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NA—59—2023

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New Pattern)

MATHEMATICS

Paper-VI

(Real Analysis-I)

(Wednesday, 13-12-2023)

Time : 2.00 p.m. to 4.00 p.m.

Time—2 Hours

Maximum Marks—40

N.B. :— (i) *All questions are compulsory.*

(ii) *Figures to the right indicate full marks.*

1. (a) *Attempt the following :*

(i) *Prove that, the set of rational number is not order-complete.* 8

(ii) *If S and T are subsets of real numbers, then show that :* 7

1. $S \subseteq T \Rightarrow S' \supseteq T'$

2. $(S \cup T)' = S' \cap T'$

Or

(b) *Attempt the following :*

(i) *Prove that, every bounded sequence with a unique limit is convergent.* 8

P.T.O.

(ii) Show that a sequence $\{S_n\}$

where $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}, n \in \mathbb{N}$ cannot converge. 7

2. (a) Attempt the following :

(i) If $\lim_{n \rightarrow \infty} a_n = l$, then prove that :

$$\lim_{n \rightarrow \infty} \left(\frac{a_1 + a_2 + \dots + a_n}{n} \right) = l. \quad 8$$

(ii) Show that the sequence $\{b_n\}$ where 7

$$b_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \dots + \frac{1}{(2n)^2} \text{ converges to zero.}$$

Or

(b) Attempt the following :

(i) If $\sum u_n$ is positive term series such that $\lim_{n \rightarrow \infty} n \left(\frac{u_n}{u_{n+1} - 1} \right) = 1$, 8

then prove that the series :

(1) converges if $l > 1$

(2) diverges if $l < 1$

(3) test fails if $l = 1$

(ii) Test the convergence of the series $\sum \frac{n^2 - 1}{n^2 + 1} \cdot x^n, x > 0$. 7

3. Attempt any *two* of the following : 10

(a) Prove that a sequence cannot converge to more than one limit.

(b) If $\lim_{n \rightarrow \infty} a_n = a$ and $a_n \geq 0$ for all n , then prove that $a \geq 0$.

- (c) Test the convergence of the series $\sum \frac{1}{n^{1+\frac{1}{n}}}$.
- (d) State the Cauchy's root test and the D'Alembert's ratio test.