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NA—63—2023

FACULTY OF ARTS AND SCIENCE

B.Sc. (Third Year) (Fifth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(CBCS/New Pattern)

MATHEMATICS

Paper–XIII

(Linear Algebra)

(Thursday, 14-12-2023) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. If U and W are two subspaces of a vector space V, then prove that U + W is a subspace of V and if z = U + W, then prove that $Z = U \oplus W$ if and only if any vector $z \in Z$, can be expressed uniquely as the sum $z = u + w$, $u \in U$, $w \in W$. 15

Or

(*a*) In a vector space V if $\{v_1, v_2, ..., v_n\}$ generates V and if $\{w_1, w_2, ..., w_m\}$ is L.I., then show that $m \leq n$. 8

P.T.O.

(*b*) Let $T : U \to V$ be a linear map, then prove that : 7

(*i*) T is one-one iff N(T) is the zero subspace.

(*ii*) If $[u_1, u_2, ..., u_n] = U$, then :

$$R(T) = [T(u_1), T(u_2), ..., T(u_n)].$$

2. Show that every real vector space of dimension p is isomorphic to V_p. Also, show that $T : P_2 \to V_3$ defined by $T(\alpha_0 + \alpha_1 x + \alpha_2 x^2) = (\alpha_0, \alpha_1, \alpha_2)$ is an isomorphism. 15

Or

(*a*) Determine the matrix $(T : B_1, B_2)$ for the linear map $T : V_3 \to V_3$ defined by :

$$T(x_1, x_2, x_3) = \left(x_1 - x_2 + x_3, 2x_1 + 3x_2 - \frac{1}{2}x_3, x_1 + x_2 - 2x_3\right)$$

$B_1 = \{e_1, e_2, e_3\}$

$B_2 = \{(1, 1, 0), (1, 2, 3), (-1, 0, 1)\}$. 8

(*b*) Let A be a square matrix of order n having k distinct eigen-values $\lambda_1, \lambda_2, ..., \lambda_k$. Let V_i be an eigenvectors corresponding to the eigenvalues λ_i, $i = 1, 2, ..., k$, then show that the set $\{V_1, V_2, ..., V_k\}$ is L.I. 7

3. Attempt any *two* of the following : 5 each

(*a*) Let S be a non-empty subset of a vector space V. Then prove that [S] the span of S, is a subspace of V.

(*b*) Let U be a subspace of a finite-dimensional vector space V, then prove that $\dim U \leq \dim V$ and equality holds only when U = V.

(*c*) Let T : U $\rightarrow$ V and S : V $\rightarrow$ W be two linear maps. Then prove that if S and T are non-singular, then ST is also non-singular and $(ST)^{-1} = T^{-1}S^{-1}$.

(*d*) Prove that any orthogonal set of non-zero vectors in an inner product space is L.I.