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NA—68—2023

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Fourth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New Course)

MATHEMATICS

Paper–IX

(Real Analysis–II)

(Thursday, 14-12-2023) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *all* questions.

(*ii*) Figures to the right indicate full marks.

1. A necessary and sufficient condition for the integrability of a bounded function f is that to every $\epsilon > 0$, there corresponds $\delta > 0$ such that for every partition p of $[a, b]$ with norm $\mu(p) < \delta$

$$U(p, f) - L(p, f) < \epsilon.$$ 15

Or

(*a*) If a function f is bounded and integrable on $[a, b]$, then the function F defined as

$$F(x) = \int_a^x f(t)dt, \quad a \leq x \leq b$$

P.T.O.

is continuous on $[a, b]$ and further more if f is continuous at a point C of $[a, b]$, then F is desirable at C and $F'(C) = f(C)$. 8

(*b*) Show that : 7

$$\int_a^t \sin x \, dx = 1 - \cos t.$$

2. Prove that the improper integral $\int_a^b \frac{dx}{(x-a)^n}$ convergence if and only if $n < 1$ and test the convergence of $\int_0^1 \frac{dx}{\sqrt{1-x^3}}$. 15

Or

(*a*) If ϕ is bounded in $[a, \infty]$, and $\int_a^\infty f dx$ is convergent at ∞, then prove that $\int_a^\infty f \phi dx$ is convergent at ∞. 8

(*b*) Prove that the integral $\int_a^\infty x^{m-1} e^{-x} dx$ is convergent if and only if $m > 0$. 7

3. Attempt any *two* of the following :

(*a*) If f is integrable on $[a, b]$, then prove that f^2 is also integrable on $[a, b]$. 5

(*b*) Prove that every continuous function is integrable. 5

(*c*) Prove that every absolutely convergent integral is convergent. 5

(*d*) Prove that the integral $\int_a^\infty f dx$ converges at ∞ if and only if for every $\in > 0$ there corresponds a positive number x_0 such that : 5

$$\left| \int_{x_1}^{x_2} f dx \right| < \in \text{ for all } x_1, x_2 > x_0.$$