This question paper contains **2** printed pages]

NA—74—2023

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New Pattern)

MATHEMATICS

Paper–VII

(Group Theory)

(Friday, 15-12-2023) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Let A be a non-empty set and let R be an equivalence relation in A. Let a and b be arbitrary elements in A. Then prove that : 15

(*i*) $a \in [a]$

(*ii*) If $b \in [a]$, then $[b] = [a]$

(*iii*) $[a] = [b]$ if and only if $(a, b) \in R$ i.e. if and only if aRb.

Or

(*a*) Prove that a non-empty subset H of a group G is a subgroup of G if and only if : 8

(*i*) $a \in H, b \in H \Rightarrow ab \in H$

(*ii*) $a \in H \Rightarrow a^{-1} \in H$ where a^{-1} is the inverse of a in G.

P.T.O.

(*b*) Show that the four fourth roots of unity namely 1, –1, i, $-i$ form a group with respect to multiplication. 7

2. Prove that the relation of congruency in a group G defined by $a \equiv b \,(\text{mod H})$ iff $ab^{-1} \in \text{H}$ is an equivalence relation. 15

Or

(*a*) A subgroup H of a group G is a normal subgroup of G if and only if the product of two right cosets of H in G is again a right coset of H in G. 8

(*b*) Show that $a \to a^{-1}$ is an automorphism of a group G if and only if G is abelian. 7

3. Attempt any *two* of the following : 5 marks each

(*a*) Show that the set I of all integers ...., –4, –3, –2, –1, 0, 1, 2, 3, 4, ... is a group with respect to the operation of addition of integers.

(*b*) Show that if a, b are any two elements of a group G, then $(ab)^2 = a^2b^2$ if and only if G is abelian.

(*c*) Prove that every proper sub-group of an infinite cyclic group is infinite.

(*d*) Show that every subgroup of an abelian group is normal.