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NA—94—2023

FACULTY OF SCIENCE

B.Sc. (Second Year) (Fourth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2023

(New Pattern)

MATHEMATICS

Paper XI

(Partial Differential Equations)

(Tuesday, 19-12-2023) **Time : 2.00 p.m. to 4.00 p.m.**

Time—Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Explain the rules for finding particular integral of the partial differential equation : 15

$$f(D, D')\, z = F(x, y),$$

when :

(*i*) $F(x, y) = e^{ax+by}$

(*ii*) $F(x, y) = \sin(ax + by)$.

Find general integral of the equation :

$$\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y.$$

P.T.O.

Or

(*a*) Explain the working rule of Lagrange's linear equation is an equation of type Pp + Qq = R. 8

Where P, Q, R are functions of x, y, z and $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

(*b*) Find the general solution of : 7

$$x(z^2 - y^2)\frac{\partial z}{\partial x} + y(x^2 - z^2)\frac{\partial z}{\partial y} = z(y^2 - x^2).$$

2. Explain Monge's method to solve the non-linear equation of second order : 15

$$\text{R}r + \text{S}s + \text{T}t = \text{V}$$

Where R, S, T, V are functions of x, y, z, p and q respectively. Solve $r = a^2 t$.

Or

(*a*) Derive the solution of wave equation : 8

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

by D' Alembert's method .

(*b*) A rod of length l with insulated sides is initially as a uniform temperature u its ends are suddenly cooled to 0ºC and kept at that temperature. Prove that the temperature function $u(x, t)$ is given by : 7

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} . e^{-\frac{c^2\pi^2 n^2 t}{l^2}}$$

where b_n is determined from the equation.

3. Attempt any *two* of the following : 10

(*a*) Solve :

$$\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x + 3y).$$

(*b*) Explain the method to solve the equation of type :

$$f(z, p, q) = 0.$$

(*c*) Solve :

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

by the method of separation of variables.

(*d*) Solve :

$$(D + D' - 2)(D + 4D' - 3)\, z = 0.$$