UNIT: I

PARTICLE PROPERTIES OF WAVES

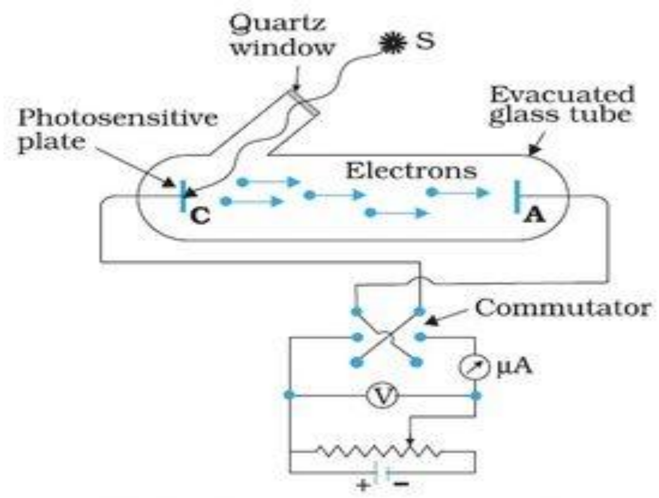
Paper: XII **Class: B.Sc. T.Y.**

1.1 PHOTOELECTRIC EFFECT:

Statements: The phenomenon of ejection of electrons from metal surface when light of suitable wavelength falls on it.

Emitted electrons are called *photoelectrons* and current due to photoelectrons are called *photoelectric current*.

Experimental Arrangement:



Window is sealed on evacuated glass tube at one end.

Ultraviolet light of suitable frequency is allow to enter through the window

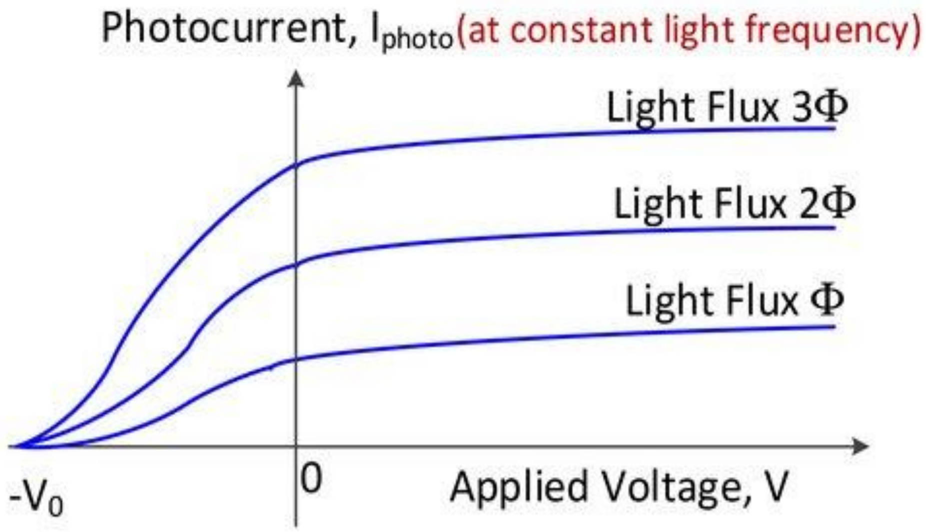
C is photosensetive plate used for photoelectric effect and *A* is used to collect emitted electrons.

Potential difference is applied across *C* and *A* using battery and potential divider.

Micro ammeter μA is used to measure the photoelectric currents and voltmeter V measures potential across C and A.

Observations: When light of suitable frequency is incident on photosensitive plate C, some photo electrons are emitted from metal surface and have enough energy to reach the collector plate despite its negative polarity and they constituents measured current.

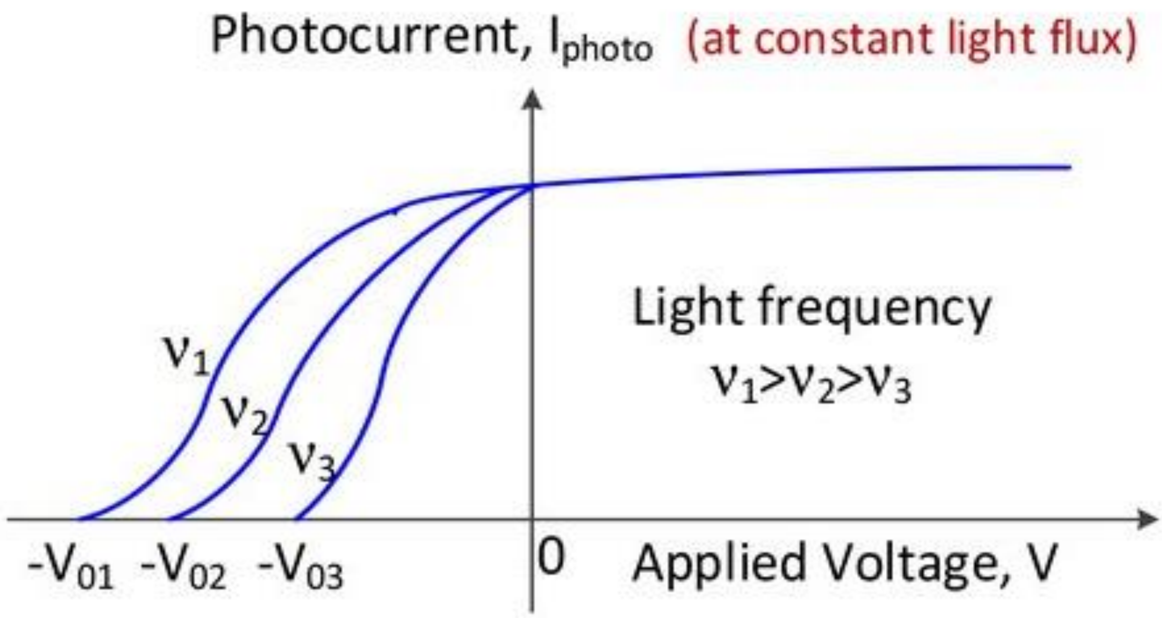
1. **Variation of photoelectric current with P.D. at constant light frequency**



With a monochromatic beam of ultraviolet radiation having particular frequency a graph with photoelectric current I on Y axis and P.D. between C and A on X axis. It is found that

1. Max. photoelectric current is directly proportional to intensity of incident light
2. Stopping potential is independent of intensity of light
3. Bright light yields more photoelectrons.

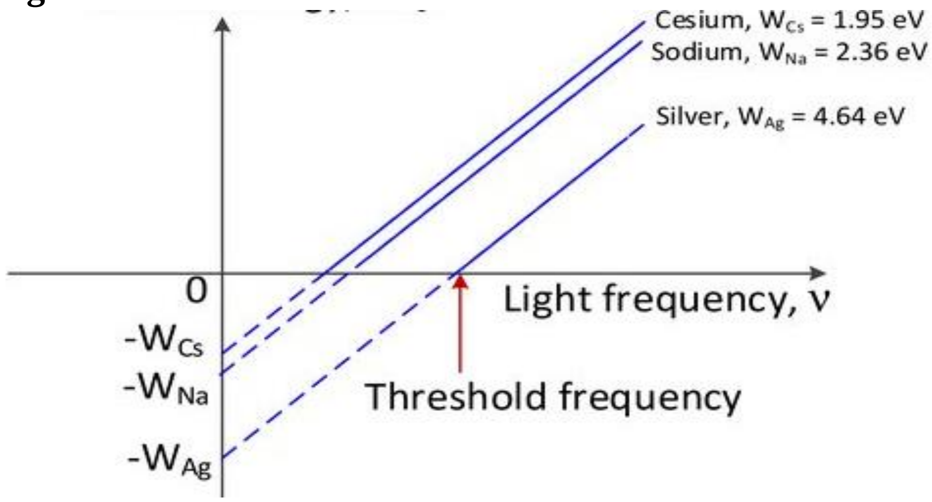
2. Variation of photoelectric current with P.D. at constant light intensity (flux)



When experiment is repeated with light of different frequency with constant intensity then the graph is as above

1. Frequency changes, stopping potential changes
2. Higher the frequency, more energy the photoelectrons have.
3. Blue light result in faster electrons than red light.

3. Variation of max.KE of photoelectron with frequency of incident light



1. **Threshold frequency (υo)** is that frequency below which no photoelectric effect is observed
2. Minimum energy required for electron to escape is **work function (ϕ)**
3. Greater the **work function (ϕ),** more energy is needed for ejection of electron and higher the threshold frequency for photoelectric emission to occur.
4. Photoelectric effect: $h\upsilon = KE_{max} + \phi$,

1.2 QUANTUM THEORY OF LIGHT:

Quantum theory of light was proposed by Albert Einstein
The experimental observations follow directly from Einstein's hypothesis.

1) The energy in light is not spread out over wave fronts but is concentrated in small packets, or photons .Each photon of frequency υ has energy $h\upsilon$ the same as Planck's quantum energy. Because of energy of light is concentrated in photons and not spread out, there should be no delay in emission of photoelectrons.
2) All photons of frequency υ have same energy, so changing intensity of monochromatic light beam will change the number of photoelectrons but not their energies.
3) Higher the frequency υ , the greater the photon energy $h\upsilon$ and so more energy the photoelectrons have.

Critical Frequency (υo): The minimum frequency of incident light below which no photoelectrons are emitted called critical frequency.

Work Function (ϕ): The minimum energy for an electron to escape from a particular metal surface called work function of metal.

ϕ is related to υo by

$$\phi = h\upsilon_0 \quad \text{---------(1)}$$

Greater the work function of metal, more the energy is needed for an electron to leave its surface and higher the critical frequency for photoelectric effect to occurs.

According to Einstein, photoelectric effect in a given metal should obey the equation

$$h\upsilon = KE_{max} + \phi$$

$h\upsilon$ is photon energy

KE_{max} is maximum kinetic energy of photo electrons

But ϕ is work function and $\phi = h\upsilon_o$

$h\upsilon = K\,E_{Max} + h\upsilon_o$ and

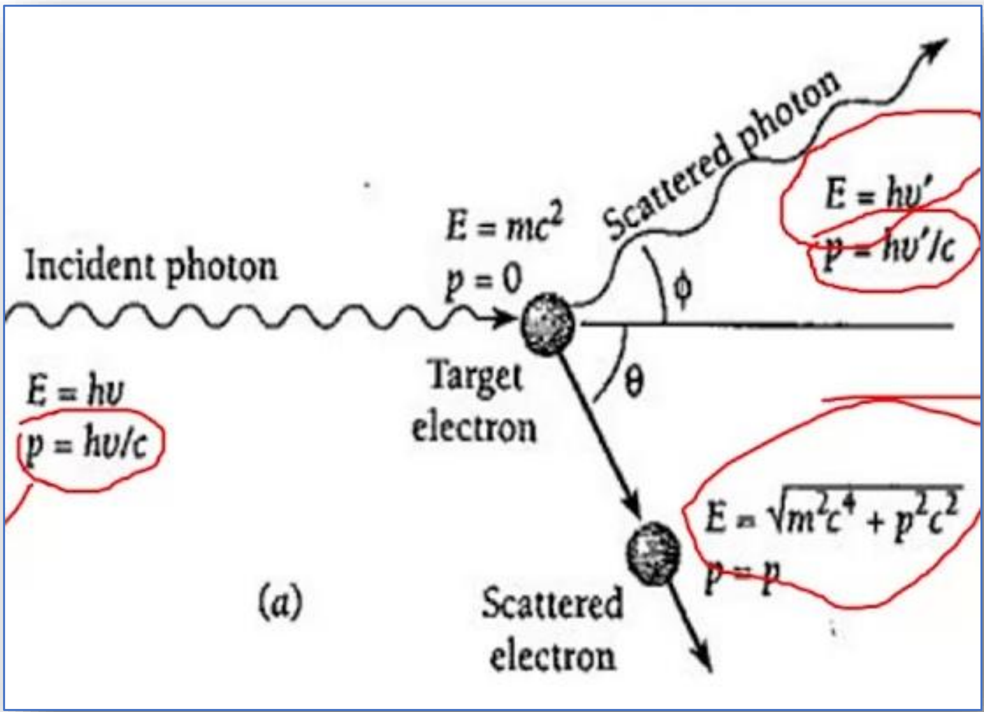
$$\boldsymbol{K\,E_{Max} = h\upsilon - h\upsilon_o = h(\upsilon - \upsilon_o)}$$

1.3 COMPTON EFFECT:

Scattering of photon by an electron

According to quantum theory of light, photons behaves like particles except for their lack of rest mass.

For that we consider a collision between photon and electron



Consider an X ray photon strikes an electron which is at rest and scattered away from its original direction of motion while electron receives an impulse and begins to move.

During collision photon looses an amount of energy that is same as kinetic energy gained by electron.

If initial photon has energy υassociated with it, scattered photon has the lower frequency υ′

Before Collision

Energy associated with initial photon of frequency υ.is

$$E = h\upsilon$$

After collision

Energy associated with scattered photon of frequency $\upsilon'$is

$$E = h\upsilon'$$

During collision

Photon looses energy electron receives that one

Loss in photon energy = Gain of KE of electron

$h\upsilon - h\upsilon'$= KE -------------(1)

Momentum of massless particle related to its energy by

E=pc -----------------------------(2)

Photon momentum

p=E/c= $h\upsilon$/c -------------------(3)

As momentum is vector quantity and in collision momentum must be conserved in each of two mutually perpendicular directions

Initial photon momentum = $h\upsilon/c$

Scattered photon momentum = $h\upsilon'$/c

Initial electron momentum = 0

Final electron momentum = p

In original direction

Initial momentum = final momentum

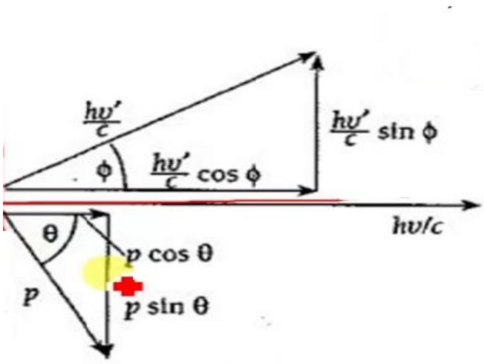
$$\frac{h\upsilon}{c} + 0 = \frac{h\upsilon'}{c}\cos\phi + p\cos\Theta \text{-----(4)}$$

Perpendicular direction

Initial momentum=final momentum

$$0 = \frac{h\upsilon'}{c}\sin\phi + p\sin\Theta \text{-------------(5)}$$

Angle ϕ is between the direction of the initial and scattered photons, and Θ is that between the direction of the initial photon and the recoil electron.



From above equations the for obtaining the wavelength difference between initial and scattered photon

Multiplying eq.(4) and (5) by c and rewriting

$pc\ cos\theta = hv - hv'cos\phi$ ----(6)

$pc\ sin\theta = hv'sin\phi$ ----- ---(7)

Squaring and adding eq. (6) and (7) θ is eliminated

$p^2c^2 = (hv)^2$-2$(hv)(hv')$cosϕ+$(hv')^2$------(8)

Total energy of particle is

E = (K.E.)+m_0c^2 -------(9)

$E = \sqrt{m_0^2c^4 + p^2c^2}$ ------(10)

From (9) and (10)

$(K.E.+m_0c^2)^2$ =$m_0^2c^4 + p^2\ c^2$

$p^2c^2 = (K.E.)^2$+2$m_0c^2(K.E)$

But KE= $h\upsilon - h\upsilon'$

We have

$p^2c^2 = (h\upsilon)^2$-2$(h\upsilon)(h\upsilon')$ +$(h\upsilon')^2$+ $2m_0c^2(h\upsilon - h\upsilon')$-----(11)

Substituting p^2c^2in eq.(8)

$2m_0c^2(h\upsilon - h\upsilon') = 2(h\upsilon)(h\upsilon')$(1-cos$\phi$) -------------------(12)

Dividing eq. (12) by 2 h^2c^2

$\frac{m_0c}{h}(\frac{\upsilon}{c} - \frac{\upsilon'}{c}) = \frac{\upsilon}{c} - \frac{\upsilon'}{c}$(1-cos$\phi$)

Since $\frac{\upsilon}{c} = \frac{1}{\lambda}$ and $\frac{\upsilon'}{c} = \frac{1}{\lambda'}$

$\frac{m_0c}{h}(\frac{1}{\lambda} - \frac{1}{\lambda'}) = \frac{1-cos\phi}{\lambda\lambda'}$

COMPTON EFFECT

$\lambda' - \lambda = \frac{h}{m_0c}$ (1-cosϕ) ---------------(13)

$\frac{h}{m_0c} = \lambda_c$ Is Compton Wavelength

CONCLUSION:

1) Compton wavelength gives the scale of wavelength change of incident photon
2) Greatest wavelength change occurs when $\boldsymbol{\phi}$ =180 degree which is twice λ_c
3) Maximum wavelength observed in X-Rays for visible it is less than 0.01%
4) X-Rays lose energy when they pass through the matter
5) Compton effect gives conformation to photon model

1.4 de-BROGLIE WAVES:

De-Broglie Hypothesis: Moving body behaves in certain ways as though it has a wave nature.

Both corpuscle and wave concept at same time.

Corpuscle and wave can not be independent

Parallelism between motion of corpuscles and propagation of associated wave

A photon of light of frequency (ν) has the momentum

$p = \frac{h\nu}{c} = \frac{h}{\lambda}$ Since c= $\nu\,\lambda$

Wavelength of a photon is specified by its momentum

Photon wavelength:

$\lambda = \frac{h}{p}$ -----------(1)

Eq.(1) is general one. Applies to material particles as well as photon

The momentum of a particle of mass (m) and velocity (ν) is

$$p = mv$$

This is **de-Broglie wavelength**

$\lambda = \frac{h}{mv}$ -----------(2)

Greater particles momentum, shorter its wavelength

'm' is relativistic mass

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Find de Broglie wavelength of

a) A 46 g golf ball with velocity of 30m/s

b) An electron with velocity of 10^7 m/s

Given: mass of golf ball $m = 0.046$ kg

velocity of golf ball $v = 30$ m/s

mass of electron $m = 9.1 \times 10^{-31}$ kg

Velocity of electron $v = 10^7$ m/s

Solution:

a) Since velocity of golf ball $v \ll c$, $m = m_0$

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10-34}{(0.046)\times 30} = 4.8 \times 10^{-34} \text{ m}$$

The wavelength of golf ball is so small compared with its dimension that we would not expect to find any wave aspect with its behaviour

Solution:

b) Since velocity of electron $v \ll c$, $m = m_0$

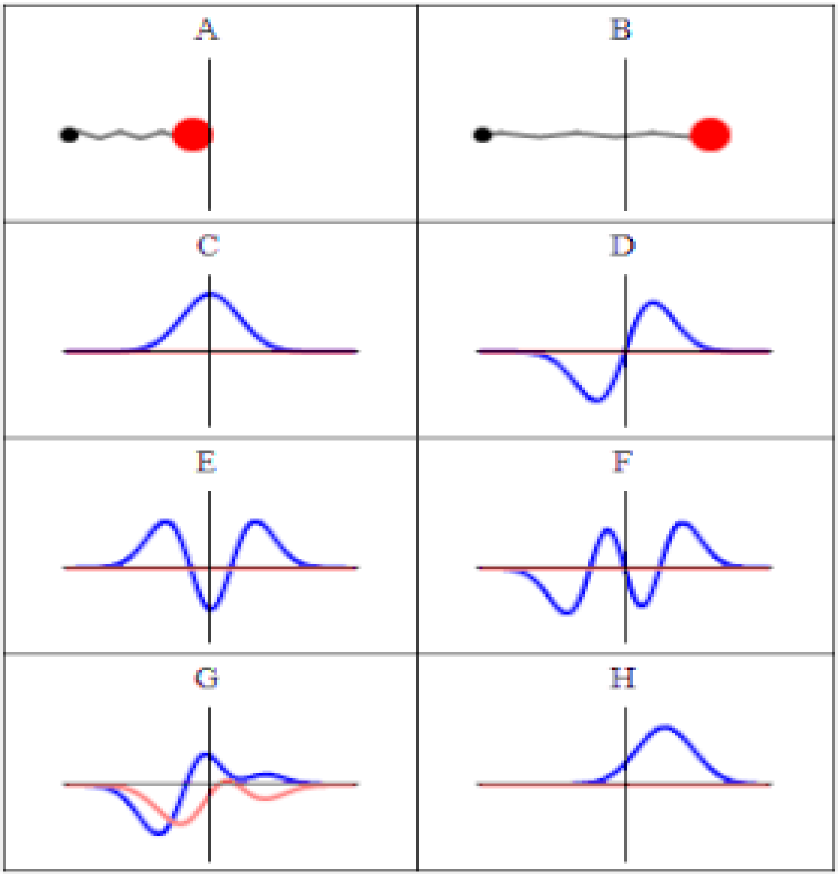
$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10-34}{(9.1\times 10-31)\times 107} = 7.3\times 10^{-11} \text{ m}$$

The wavelength of electron is comparable with its dimension i.e. (radius of hydrogen atom 5.5×10^{-11}) we could expect the wave character of moving electron

1.5 WAVE FUNCTION:

Wave function(ψ): The quantity whose variations make up matter waves called wave function.

The value of **(ψ)** associated with a moving body at a particular point x,y, z in space and at a time t is related to likelihood finding body there at the time.



Wave function is usually complex with both real and imaginary part. Probability

Wave function(ψ): No direct physical significance. The simple reason is that probability of finding object lies between 0(Object is definitely not there) and 1(Object is definitely there). Intermediate probability, say 0.3 means 30% chance a of finding the object. But amplitude of wave is negative also Negative probability say, -0.3 meaningless.

Hence $\boldsymbol{\psi}$ by itself can not be an observable quantity.

The square of absolute value of wave function $|\boldsymbol{\psi}^2|$ known as probability density is used. Large value of $|\boldsymbol{\psi}^2|$ means strong probability body's presence. While small value of $|\boldsymbol{\psi}^2|$ means slight probability of its presence.

1.6 de- BROGLIE WAVE VELOCITY:

de Broglie wave associated with moving body, has same velocity as that of body

Let Vp is de Broglie wave velocity $Vp=\nu\lambda$

But $\lambda = \frac{h}{mv}$ And $E = h\nu$

for relativistic total energy $E = mc^2$

$\therefore h\nu=mc^2$

$\nu = \frac{mc^2}{h}$

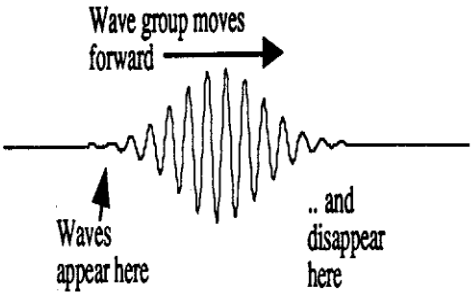
de-Broglie wave velocity (Phase velocity) is

$Vp=\nu\lambda = \frac{mc^2}{h}$ x $\frac{h}{mv} = \frac{c^2}{v}$

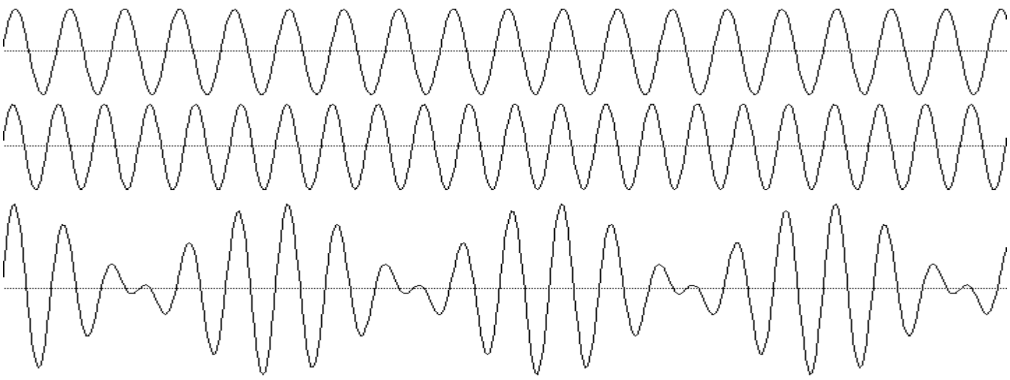
Particle velocity v must be less than velocity of light c, de-Broglie wave always travels faster than light. This is unexpected. To understand this unexpected result, we must look into the distinction between Phase velocity (wave velocity) and group velocity.

1.7 PHASE AND GROUP VELOCITIES:

Amplitude of de-Broglie waves that correspond to a moving body reflect the probability of finding body there at that time. The wave representation of moving body corresponds to **wave packet** or **wave group** as shown in fig.



A familiar example of how wave groups come in the case of formation of beats. When two sound waves of same amplitude but slightly differ in frequencies superpose with each other we hear the sound which has frequency equal to average of two original frequencies and its amplitude rise and fall periodically. The production of beat is shown in fig.



(Beats are produced by superposition of two waves of slightly differ frequencies)

Group velocity of a wave is the velocity with which the variations in the shape of the wave's amplitude (modulation or envelope of the wave) propagate through space.

The group velocity is defined by the equation

$$v_g = \frac{\delta\omega}{\delta k}$$

Where v_g is group velocity and ω is the wave's angular frequency;

k is the wave number.

- Phase velocity is the rate at which the phase of the wave propagates in space.
- This is the velocity at which the phase of any one frequency component of the wave will propagate.
- You could pick one particular phase of the wave and it would appear to travel at the phase velocity.
- The phase velocity is given in terms of the wave's angular frequency(ω) and wave vector k by

$$v_{\mathrm{p}} = \frac{\omega}{k}$$

Mathematical description of wave group:

Consider wave group arises from two waves of same Amplitude A but differ Δω in angular frequency and Δk in wave number.

The original waves are represented by

$$y_1 = A\cos(\omega t - kx)$$

$$y_2 = A\cos[(\omega + \Delta\omega)t - (k + \Delta k)x]$$

The resultant displacement at any time t and any position x is sum of y1 and y2 and using identity

$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha\text{-}\beta)$ and

the relation

$\cos(\text{-}\theta) = \cos\theta$

$$y = y_1 + y_2$$

$$y = A\cos(\omega t - kx) + A\cos[(\omega + \omega)t - (k + dk)x]$$

$$y = 2A\cos\left[\frac{(2\omega + \Delta\omega)t}{2} + \frac{(2k + \Delta k)x}{2}\right]\cos\left[\frac{(\Delta\omega)t}{2} - \frac{(\Delta k)x}{2}\right]$$

$$with \quad d\omega << \omega, dk \ << k$$

$$y \cong 2A\cos[\omega t - kx]\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right] - - - - - - (1)$$

Equation1 represent a wave of angular velocity ω and wave number k which has superimposed upon it a wave (the process is called modulation) of angular velocity $\frac{\Delta\omega}{2}$ and wave number $\frac{\Delta k}{2}$

The effect of modulation is thus to produce successive wave groups

Phase velocity

wave velocity of carrier : $v_p \quad \frac{\omega}{k}$

Group velocity

wave velocity of envelope : $v_g = \frac{\Delta\omega}{\Delta k}$

for more than two wave contiributions: $v_g = \frac{d\omega}{dk}$

Angular frequency and Wave number of de-Broglie wave associated with body of rest mass m_0 moving with velocity v is

$$\omega = 2\pi\nu$$

$$\omega = 2\pi \frac{E}{h}$$

$$\omega = 2\pi \frac{mc^2}{h}$$

Angular frequency of de-Broglie wave is

$$\omega = 2\pi \frac{m_0 c^2}{h\sqrt{1-\frac{v^2}{c^2}}}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi m v}{h}$$

Wave number of de-Broglie waves

$$k = \frac{2\pi m_0 v}{h\sqrt{1-\frac{v^2}{c^2}}}$$

Both ω and k are functions of body's velocity v

Group velocity of de-Broglie wave associated with body

$$v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv}$$

$$d\omega/dv = \frac{2\pi}{h}\frac{m_0 v}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

$$dk/dv = \frac{2\pi m_0}{h}\frac{1}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

$$\Rightarrow v_g = v$$

De-Broglie group associated with moving body travels with same velocity that of body.

1.8 G.P.THOMSON EXPERIMENT:

(Conformation of matter waves)

Construction:

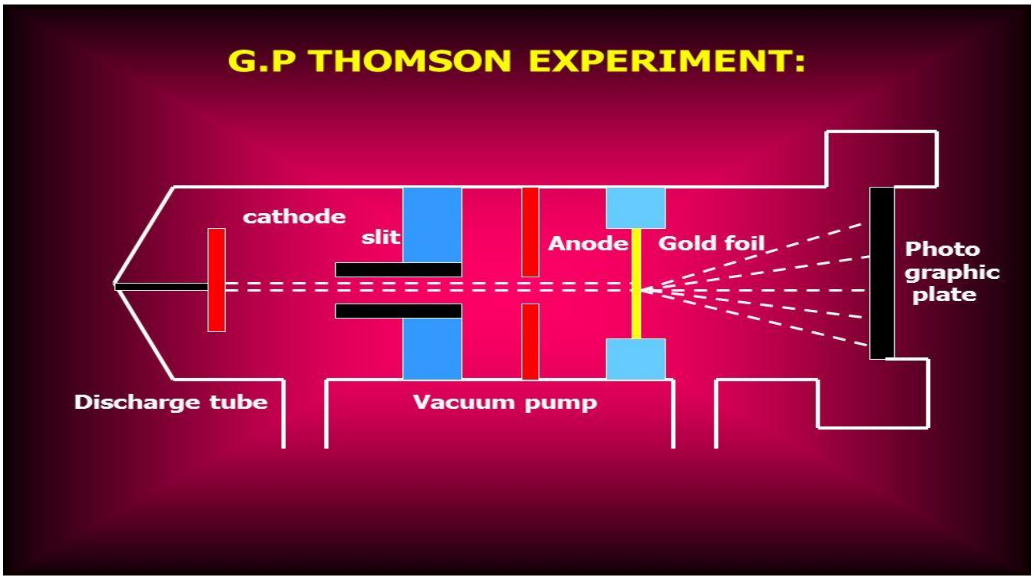
A beam of cathode ray is produced in a discharge tube by means of induction coil.

Electrons passing through a fine hole (slit) are incident on a thin gold foil**.**

The emergent beam of electrons is received on a photographic plate p

The visual examination of pattern is made possible by fluorescent screen.

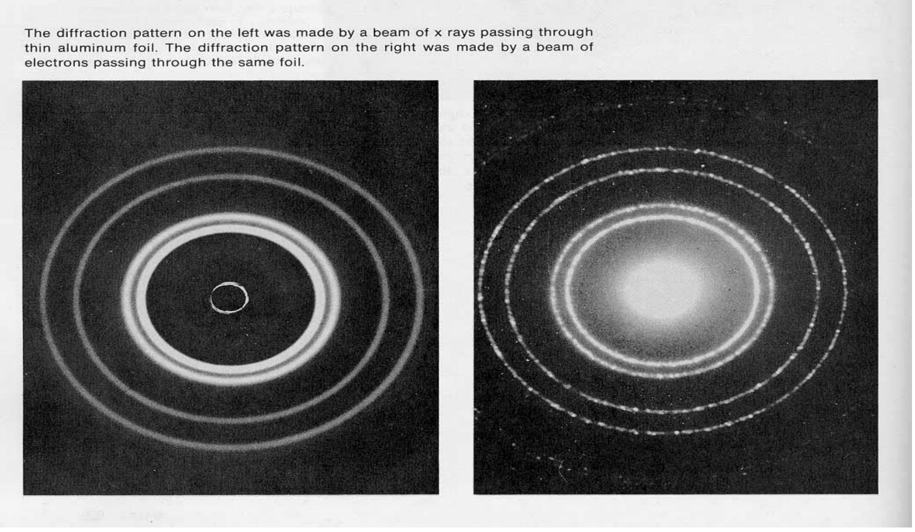
A very high vaccum is maintained in a camera part while air is allowed to leak into the discharge tube.



Procedure:

A beam of electron of known velocity is made to fall on the photographic plate, after traversing the thin gold foil.

When plate is developed a symmetric pattern consisting of concentric rings about a center spot is obtained.



It is similar to produced by X rays in powdered crystal method.

When cathode rays are deflected by magnetic field, pattern also shift correspondingly. If foil is removed pattern disappear

Demonstration:

This experiment demonstrate that electron beam behaves as wave since diffraction pattern is produced only by waves

Formula:

$$\lambda = 12.27/\sqrt{V}$$

V is accelerating voltage.

1.10 UNCERTAINTY PRINCIPLE:

Uncertainty principle states that the product of uncertainty Δx in the position of an object at same instant and uncertainty Δp in its momentum components in the x direction at the same instant is equal to or greater than h/4π or ħ /2

i.e. $\Delta p . \Delta x \geq \frac{h}{4\pi}$

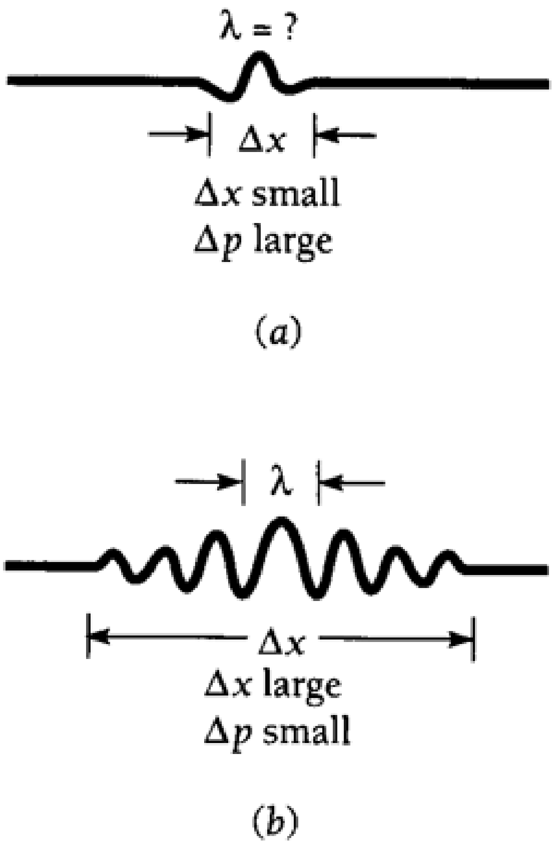


Figure 3.12 (a) A narrow de Broglie wave group. The position of the particle can be precisely determined, but the wavelength (and hence the particle's momentum) cannot be established because there are not enough waves to measure accurately. (b) A wide wave group. Now the wavelength can be precisely determined but not the position of the particle.

Applications of Heisenberg's Uncertainty Principle:

1) Electron does not exist in nucleus:

The nucleus radius is of the order of 10^{-14} m.

If we assume electron is confined in a nucleus the uncertainty in its position is $\Delta x=2 \times 10^{-14}$ equal to diameter

$\hbar= 6.63 \times 10^{-34} /6.28= 1.05 \times 10^{-34}$ Js

Uncertainty in Δp is

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.54X10-34}{2\ X10-14}$$

$$=5.275 \times 10^{-21} \text{ kg m/s}$$

If this is the uncertainty in the momentum of electron.

Momentum must be at least comparable in magnitude

$p=5.275 \times 10^{-21}$ kg m/s

K.E of electron of mass m maximum have

$$E = \frac{p^2}{2m} = \frac{(5.275 \text{ x}10{-}21)^2}{2\ x9.1x10-31}$$

=97 Mev

If electron is present in nucleus K.E. =97 Mev

But

Experimentally it is found that it is only 4 Mev

Electron is not in nucleus