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B.Sc. S.Y. Semester – IV
Paper – X

(Ring Theory)
Unit – I

1. Ring is an algebraic structure equipped with ______ binary operations.

a) One b) Two c) Three d) Four

2. The algebraic structure $(R, +, \cdot)$ is called a ring, if
a) Addition is associative and Commutative.
b) There exist additive identity and additive inverse.
c) Multiplication is associative and Multiplication is distributive with respect to addition.
d) All of the above are satisfied.

3. If the ring $(R, +, \cdot)$ possesses the multiplicative identity i.e. $1 \cdot a = a \cdot 1 = a \;\forall a \in R$ the Ring is called

a) Commutative ring b) Ring without unity
c) Ring with unity d) none of these.

4. If in a ring $(R, +, \cdot)$, $a \cdot b = b \cdot a \;\forall a, b \in R$ then ring is called

a) Commutative ring b) Ring without unity
c) Ring with unity d) none of these.

5. If R is a ring, then (for all $a,b,c \in R$) which of the following is true?

a) $a.0 = 0.a = 0$
b) $a.(-b) = -(a.b) = (-a).b$ and $(-a).(-b) = ab$
c) $a(b-c) = ab - ac$ and $(b-c)a = ba - ca$
✓d) All of the above.

6. The set R Consisting of a single element 0 with two binary operations defined by $0+0 = 0$ and $0.0 = 0$ is a ring. This ring is called

a) null ring b) zero ring
✓c) both a) & b) d) none of a) & b)

7. which of the following is a ring?

a) set of integers (I)
b) set of rationals (Q)
c) set of real numbers (R)
✓d) All of above.

8. Which of the following is not a Commutative ring?

a) set I of integer b) set Q of rationals
✓c) The set M of all $n \times n$ matrices with their elements as real numbers.
d) set $\mathbb{R}$ of real numbers

9. A non-zero element 'a' of a ring R is called a ______ if there exist an element $b \neq 0 \in R$ such that either $ab = 0$ or $ba = 0$.
a) zero divisor b) divisor of zero
✓c) both a) & b) d) none of a) & b).

10. If in a ring R, $ab = 0 \Rightarrow a = 0$ (or) $b = 0$ then R is called ______
a) Ring with zero divisor
✓b) Ring without zero divisor c) unit ring
d) none of these.

11. If in a ring R, for $a \neq 0 \in R$ and $b \neq 0 \in R$, $a \cdot b = 0$ then R is called ______.
✓a) Ring with zero divisor.
b) Ring without zero divisor c) unit ring
d) none of these.

12. Cancellation law holds in a ring R if ______
a) $a \neq 0$ $ab = ac \Rightarrow b = c$ where $a, b, c \in R$
b) $a \neq 0$, $ba = ca \Rightarrow b = c$ where $a, b, c \in R$
✓c) both a) & b) d) none of a) & b).

13. A ring R is without zero divisors if and only if
✓a) Cancellation laws hold in R.
b) Cancellation laws doesnot hold in R.
c) for some $a \neq 0 \in R$ & $b \neq 0 \in R$, $a \cdot b = 0$
d) none of these.

14. A ring is called an integral domain if it
a) is Commutative b) has unit element
c) is without zero divisors ~~d~~ All of these.

15. If R is a ring with unity, then an element $a \in R$ is called invertible, if there exist $b \in R$ such that
a) $ab = 1 = ba$ b) $ab = 0 = ba$
c) $ab = ba \neq 1$ d) none of these.

16. Invertible elements of ring of integers is
a) 1 & -1 b) only 1 c) only -1 d) 0

17. A ring R with at least two elements is called a field if it,
a) is Commutative b) has unity
c) is such that each non-zero element possesses multiplicative inverse ~~d~~ All of these.

18. A ring with at least two elements is called a division ring or skew field if it
a) has unity b) is such that each non-zero element possess multiplicative inverse
c) both a) & b) d) none of a) & b)

19. Every field is
a) Integral domain b) skew field
c) both a) & b) d) none of a) & b)

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20. A skew field has
a) divisors of zero b) no divisors of zero
c) $a \cdot b = 0 \Rightarrow a = 0$ or $b = 0$ ✓d) both a) & c)

21. Every ________ is a field.
✓a) finite integral domain b) skew-field
c) infinite integral domain d) integral domain

22. Which of the following is a field?
a) $\mathbb{Q}$ b) $\mathbb{R}$ c) $\mathbb{C}$ ✓d) All of these.

23. Which of the following is not a field?
✓a) I b) $\mathbb{R}$ c) $\mathbb{C}$ d) $\mathbb{Q}$

24. Commutative division ring is ________.
a) An integral domain b) field
✓c) both a) & b) d) none of these.

25. Which of the following are integral domains?
a) $(I, +, \cdot)$ b) $(R, +, \cdot)$
c) $(\mathbb{Q}, +, \cdot)$ ✓d) All of these.

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Unit - II

1. A ring R is said to be isomorphic to another ring R' if there exist a one-one mapping f of R onto R' such that ∀ a, b ∈ R

a) $f(a+b) = f(a) + f(b)$ b) $f(a.b) = f(a).f(b)$
c) both a) & b) d) none of these.

2. If f is a isomorphism of a ring R onto a ring R' then
a) the image of the zero of R is the zero of R'
b) the image of the negative of an element of R is the negative of the image of that element i.e, $f(-a) = -f(a)$ ∀ $a \in R$
c) If R is Commutative ring, then R' is also a Commutative ring.
d) All of the above.

3. If f is a isomorphism of a ring R onto a ring R' then
a) If R is without zero divisors, then R' is also without zero divisors.
b) If R is with unit element, then R' is also with unit element
c) If R is a field then R' is also field.
d) All of the above.

4. If R is any ring then {0} & R are called ______
a) proper subrings of R b) improper subring of R
c) Sometimes proper and sometimes improper
d) none of these.

5. If R is any ring, then subrings except $\{0\}$ & R are called ___:
a) proper subrings of R
b) Improper subrings of R.
c) depends upon subrings.
d) none of these.

6. Let R be a ring with zero element 0 and suppose there exists a positive integer n such that $na = a + a + \cdots +$ upto n term $= 0$ for every $a \in R$. The smallest such positive integer n is called the

a) Characteristics of R b) Degree of R.
c) order of R d) none of these.

7. The Characteristics of $I_6 = \{0,1,2,3,4,5\}$ is
a) 5 b) 6 c) 7 d) 8

8. The Characteristics of the ring I_n ; $n \in N$ is
a) n b) $n+1$ c) $n+2$ d) $n+3$

9. The Characteristics of set of integers I is
a) 1 b) 0 c) 2 d) 3

10. Which of the following rings have Characteristics zero?

a) I b) Q c) R d) All of these.

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11. The Characteristic of a ring with unity is 0 or $n>0$ according as the unity element 1 regarded as a member of the additive group of the ring has the order

a) zero b) n c) a) or b) d) none of these.

12. The characteristic of an integral domain is 0 or $n>0$ according as the order of any non-zero element regarded as a member of the additive group of the integral is

a) 0 b) n c) either 0 or n d) None of these.

13. Each non-zero element of an integral domain D regarded as a member of the additive group of D, is of the

a) same order b) different order
c) Can be both a) or b) d) None of these.

14. The characteristic of an integral domain is ________

a) prime number b) zero
c) either a) or b) d) one

15. The characteristics of field is ________
a) zero b) one
c) prime number d) either a) or c)

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(16) A ring R is said to be imbedded in a ring R' if there is a subring S' of R' such that

a) R is isomorphic to S'
b) R is isomorphic to S
c) R is isomorphic to R'
d) S' is isomorphic to R'

(17) Any ring R without a unity element can be imbedded in a ring ______.

a) with unity b) without unity
c) both a) & b) d) None of these.

(18) A commutative ring without zero divisors can be imbedded in a ______

a) any ring b) a field ~~c) both~~
c) both a) & b) d) None of these.

(19) Every integral domain can be imbedded in a ______.

a) Integral domain b) field
c) both a) & b) d) None of a) & b)

(20) If K is any field which contains an integral domain D, then K contains subfield isomorphic to the

a) D b) quotient field of D
c) both a) & b) d) None of a) & b)

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(21) A non-empty subset S of a ring R is said to be left ideal of R if S satisfy

a) S is a subgroup of R with respect to addition.
b) $r.s \in S \ \forall \ r \in R$ & $\forall \ s \in S$
c) both a) & b) ✓
d) None of a) & b)

(22) A non-empty subset S of a ring R is said to be a right ideal if S satisfy

a) S is a subgroup of R under addition.
b) $s.r \in S \ \forall \ r \in S$ & $s \in S$.
c) both a) & b) ✓
d) None of a) & b).

(23) A non-empty subset S of a ring R is said to be ideal if S is:

a) left ideal b) Right ideal
c) both a) & b) ✓ d) None of a) & b)

(24) If R is any ring ~~then~~ R and {0} are called

a) proper ideals b) Improper ideal ✓
c) both a) & b) d) None of a) & b)

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(25) If R is a ring then any ideal except R & {0} are called ______.

a) proper ideals b) Improper ideals
c) both a) & b) d) None of a) & b)

(26) The intersection of any two ~~ideals~~ left (right) ideal is ______.

a) left (right) b) right (left)
c) ideal d) All of these.

(27) The left ideal generated by the union $I_1 \cup I_2$ of two left ideals is the set ______.

a) $I_1 - I_2$ b) $I_1 + I_2$ c) I_1 d) I_2

(28) A ideal S of a ring R is said to be a principal ideal if there exists an element $a \in S$ such that any ideal T of R containing 'a' also contains

a) S b) R c) T d) None of these.

(29) If R is a commutative ring without zero divisors and with unity and if every ideal S in R is of the form $S = (a)$ for some $a \in S$. then R is ______
a) field b) principal ideal ring c) division ring
d) None of these.

(30) The ring of integers is ______
a) principal ideal ring b) field
c) division ring d) None of these.

Unit - III

1) If $0 \neq a \in R$ then a is said to divide $b \in R$ if there exists an element $c \in R$ such that

a) $b = ac$ b) $b^2 = ac$
c) both a) & b) d) None of a) & b)

2) Units of a Ring R are those elements which possesses

a) Additive inverses only
b) Multiplicative Inverse only
c) Multiplicative Inverse.
d) None of these.

3) The units of the integral domain of Gaussian Integers are

a) ± 1 b) $\pm i$
c) both a) & b) d) None of a) & b)

4) An element 'a' is said to be an associate of Commutative ring with unity R if ___

a) $a = ub$ b) $a = b$
c) both a) & b) d) None of a) & b)

5) If R is a Commutative ring with unity, then the relation in R defined by "a is an associate of b" is

a) Reflexive only b) Symmetric only
c) Transitive only d) An equivalence Relation

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6) Let R be a commutative ring. If $a, b \in R$ then $0 \neq d \in R$ is said to be greatest common divisor of a & b if

a) $d|a$ & $d|b$ b) whenever $c|a$ & $c|b$ then $c|d$

✓ c) both a) & b) d) None of a) & b)

7) Which of the following is true if $f(x)$ & $g(x)$ are two non-zero polynomials over an arbitrary ring R.

a) $\deg[f(x)+g(x)] \leq \max[\deg f(x), \deg g(x)]$

b) $\deg[f(x) \cdot g(x)] \leq \deg f(x) + \deg g(x)$

c) None of a) & b).

✓ d) both a) & b)

8) If R is an arbitrary ring and R' is the set of constant polynomials over ~~in~~ $R[x]$ then

✓ a) R' is isomorphic to R
b) $R' \subseteq R$
c) $R' = R$
d) None of these.

9) If D is an integral domain then $D[x]$ is ______.

a) Division ring b) Integral domain
c) field d) All of these.

10) If R is an integral domain with unity element then which of the following is true?

a) Every element in $R[x]$ is unit in R
b) Every unit in $R[x]$ is unit in R.
c) both a) & b)
d) None of a) & b).

11) If F is field then the set $F[x]$ is ______.

a) Field b) Division ring
c) Integral domain d) None of these.

12) If R is an integral domain then $R[x_1, \dots, x_n]$ is ______.

a) field b) division ring
c) integral domain d) none of these.

13) If F is a field then $F[x]$ is a ______.

a) Integral domain b) field
c) division ring d) None of these.

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14) Let D be an integral domain with unity element 1. A non-zero non-unit element $a \in D$ having only trivial divisors, is called ______

a) prime element b) irreducible element
c) Reducible element d) both a) & b)

15) An element $0 \neq b \in D$ having proper divisors where D is an integral domain is called ______

a) Reducible element b) prime element
c) Composite element d) both a) & c).

16) Two polynomials $f(x)$ & $g(x) \in F[x]$ are said to be relatively prime if their greatest Common divisor is ______

a) prime element of F
b) Unity element of F.
c) both a) & b)
c) None of a) & b)

17) Let $f(x), g(x) \neq 0$ be any two polynomials domain $F[x]$, over the field F. then there exist uniquely two polynomials $q(x)$ & $r(x)$ in $F[x]$ such that

a) $f(x) = q(x) \cdot g(x) + r(x)$ where $r(x)=0$ or $\deg r(x) < \deg g(x)$
b) $g(x) = r(x) \cdot q(x) + g(x)$ where $g(x) = 0$

c) both a) & b) d) None of a) & b)

18) A polynomial ring / domain $F[x]$ over a field F is a ______.

a) field b) principal ideal ring
c) Integral domain d) both b) & c)

19) Which of the following is not a principal ideal ring?

a) $I[x]$ b) $Q[x]$ c) $R[x]$ d) $Z_7[x]$

20) Let F be a field and $f(x)$ and $g(x)$ be any two polynomial in $F[x]$, not both of which are zero then $f(x)$ & $g(x)$ have greatest common divisor $d(x)$ which can be expressed in the form

a) $d(x) = m(x) \cdot f(x) + n(x) \cdot g(x)$
where $m(x), n(x) \in F[x]$

b) $d(x) = \frac{m(x)}{f(x)} + \frac{n(x)}{g(x)}$
where $m(x), n(x) \in F[x]$.

c) both a) & b)

d) None of a) & b)

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Unit - IV

1) An Integral domain R, with unity element 1 is a unique factorization domain if

a) Any non-zero element in R is either a unit or can be written as the product of a finite number of irreducible (prime) elements of R.

b) The decomposition is unique upto the order and associates of the irreducible elements.

✓c) both a) & b)
d) None of a) & b).

2) Let $f(x), g(x)$ be polynomials in $F[x]$ for a field F. If $f(x) \mid g(x).h(x)$ and the greatest common divisor of $f(x)$ and $g(x)$ is 1 then

✓a) $f(x) \mid h(x)$ b) $h(x) \mid f(x)$
c) both a) & b) d) None of a) & b)

3) If $f(x)$ is an irreducible polynomial in $F[x]$ for a field F and ~~f(x)~~ $f(x) \mid g(x).h(x)$ where $g(x), h(x) \in F[x]$ then

a) $f(x)$ divides both $g(x)$ & $h(x)$
✓b) $f(x)$ divides at least one of $g(x)$ or $h(x)$
c) $f(x)$ divides neither $g(x)$ nor $h(x)$
d) $f(x)$ divides $g(x)$ only.

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4) If S is an ideal of a ring R, then the set

$$R/S = \{S + a ; a \in R\}$$ forms a ——.

a) group only ~~b)~~ ring
c) field d) none of these.

5) A mapping f from a ring R into a ring R' is said to be a homomorphism of R into R' if

a) $f(a+b) = f(a) + f(b) \; \forall \; a, b \in R$
b) $f(a \cdot b) = f(a) \cdot f(b) \; \forall \; a, b \in R$
✓c) both a) & b)
d) None of a) & b)

6) A mapping f from a ring R onto a ring R' is said to be a homomorphism of R onto R' if

a) $f(a+b) = f(a) + f(b) \; \forall \; a, b \in R$
b) $f(ab) = f(a) \cdot f(b) \; \forall \; a, b \in R$
✓c) both a) & b)
d) None of a) & b)

7) If f is a homomorphism of a ring R into a ring R' then
a) $f(0) = 0'$ b) $f(-a) = -f(a)$
✓c) both a) & b) d) None of a) & b)

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8) Let ϕ be a homomorphic mapping of a ring R into a ring R'. Let S' be the homomorphic image of R in R', then

a) S' is a subring of R
✓b) S' is a subring of R'
c) S' is a field.
d) None of these.

9) If f is a homomorphism of R into R' then S is the kernel of f if

✓a) $S = \{x \in R : f(x) = 0'$ where $0'$ is the zero element of $R'\}$

b) $S = \{x \in R' : f(x) = 0$ where 0 is the zero element of $R\}$

c) both a) & b)
d) None of a) & b)

10) If f is a homomorphism of a ring R into a ring R' with kernel S then

a) S is an ideal of R
b) S is a left ideal of R
c) S is a right ideal of R
✓d) All of the above.

11) The homomorphism ϕ of a ring R into a ring R' is an isomorphism of R into R' if and only if ~~...~~ ______, where $I(\phi)$ denotes kernel of ϕ.

a) $I(\phi) = R$ b) $I(\phi) = R'$
✓ c) $I(\phi) = (0)$ d) None of these.

12) Suppose R is a ring, S is an ideal of R. let f be a mapping from R to R/S defined by $f(a) = S + a$ $\forall\, a \in R$. Then f is a homomorphism of ______

✓ a) R onto R/S
b) R onto R
c) R onto S
d) None of these.

13) Every homomorphic image of a ring R is isomorphic to some ______ of itself.

a) residue class ring b) quotient ring
✓ c) both a) & b) d) None of a) & b).

14) An Ideal $S \neq R$ in a ring R is said to be a maximal ideal of R if whenever U is an ideal of R such than $S \subseteq U \subseteq R$ then ______

a) $R = U$ b) $S = U$
✓ c) either $R = U$ or $S = U$ d) neither $R = U$ nor $S = U$.

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15) An Ideal S of the ring of integers I is maximal if and only if S is generated by ______

a) Any integer b) Any positive integer
c) some prime number d) None of these.

16) Let S_1 & S_2 be two ideals of a ring R and let
$S_1 + S_2 = \{s_1 + s_2 ; s_1 \in S_1 ; s_2 \in S_2\}$
then $S_1 + S_2$ is generated by

a) $S_1 \cup S_2$ b) $S_1 \cap S_2$
c) S_1 d) S_2

17) If an ideal U of a ring R Contains a unit of R then ______.

a) $U \neq R$ b) $U = R$
c) U is field d) None of these.

18) Let R be a Commutative ring with unity and a, b be two non-zero elements of R then

a) $(a) = (b)$ iff a/b
b) $(a) = (b)$ iff b/a
c) $(a) = (b)$ iff a/b and b/a
d) $(a) = (b)$ iff a/b or b/a

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19) An ideal S of Commutative ring R with unity is maximal if and only if the residue class ring R/S is ____

a) integral domain b) division ring.
✓ c) field d) none of these.

20) Let R be a ring and S an ideal in R. Then S is said to be a prime ideal of R if ab ∈ S, a, b ∈ R implies that ____

a) a is in S b) b is in S.
✓ c) either a or b is in S
d) neither a nor b is in S.

21) Let R be a Commutative ring and S is an ideal of R then R/S is an integral domain iff ____

✓ a) S is prime ideal b) S is maximal ideal
c) both a) & b) d) None of a) & b).

22) Let R be a Commutative ring with unity then

✓ a) Every maximal ideal of R is a prime ideal
b) Every prime ideal of R is a maximal ideal.
c) both a) & b).
d) None of a) & b).

23) The ring of integral is a ____
a) field ✓ b) Euclidean ring
c) both a) & b) d) None of a) & b)

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24) The ring of Guassian Integers is a

a) Euclidean ring.
b) field
c) both a) & b)
d) None of a) & b)

25) Which of the following is true?

a) Every Euchidean ring is a principal ideal ring.
b) Every principal ideal ring is a Euclidean ring.
c) both a) & b)
d) none of a) & b).