Integral Calculus
Paper No. - III.
UNIT - I

1> The following functional operations are inverse of each other
a) $y = e^x$ & $x = \log y$
b) $y = x^3+1$ & $x = (y-1)^{1/3}$

2> If $\frac{d}{dx} F(x) = f(x)$, then $F(x)$ is an integral of $f(x)$ & write $\int f(x)dx = F(x)$

3> $\int \cos x \, dx = \sin x$

4> If $\frac{d}{dx} F(x) = f(x)$ then
$\int f(x)\,dx = F(x) + C$

5> The process of finding integration is called integration

6> The fu^n which is to be integrated called integrand.

7> $\frac{d}{dx}\left(\int f(x)\,dx\right) = f(x)$

8> $\int \lambda \cos x \, d\lambda = \frac{\lambda^2}{2} \cos x + C$

9> $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

10> $\frac{d}{dx}(-\cos^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

11> $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\frac{x-a}{x+a}$ OR $\frac{1}{2a}\log\frac{a-x}{a+x}$

12> $\int a^x\,dx = a^x/\log a$

13> $\int \frac{1}{\sqrt{2x-x^2}}\,dx = \operatorname{vers}^{-1}x$

14> $\int [f_1(x) \pm f_2(x)]\,dx = \int f_1(x)\,dx \pm \int f_2(x)\,dx$

15> $\int (\cos x + x^n)\,dx = \sin x + \frac{x^{n+1}}{n+1}$

16> $\int \left(2\sin x + \frac{1}{x}\right)dx = -2\cos x + \log x$

17> The integral of product of a const. & a fun is equal to the product of const. & the integral of fun.

i.e. $\int a\,f(x)\,dx = a\int f(x)\,dx$

18> Four methods of integration:

1) substitution
2> Integration by parts
3> Decomposition into sum
4> successive reduction

19> $\int x\cos x^2\,dx = \frac{1}{2}\sin x^2$

20) $\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b)$

21) $\int f(ax+b)\,dx = \frac{1}{a} F(ax+b)$

22) $\int \sin^2 x . \cos x\,dx = \frac{1}{3}\sin^3 x$

23) $\int \frac{f'(x)}{f(x)}\,dx = \log f(x)$

24) $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

25) $\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\log(x^2+1)$

26) Integral of product of two fu^ns
= First fu^n x integral of second
- integral of (diff. coeff. of First x
integral of second)

27) $\int f_1(x) . f_2(x)\,dx$ [Int. by parts]

$= f_1(x)\int f_2(x)\,dx - \int\left(f_1'(x) . \int f_2(x)\,dx\right)dx$

28) $\int x\cos x\,dx = x\sin x + \cos x$

29) $\int \log x\,dx = x\log x - x$ OR $x\log\left(\frac{x}{e}\right)$

30) $\int e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x)$

31) $\int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x . \cos x + \left[\frac{n-1}{n}\right] \int \sin^{n-2} x \, dx$

32) A formula which connects an integral with another in which the integrad is same type, but is of lower degree or order is called reduction formula.

33) The fraction
$$\frac{a_0 x^m + a_1 x^{m-1} + \dots + a_m}{b_0 x^n + b_1 x^{n-1} + \dots + b_n}$$
in which $a_0, a_1, a_2 \dots b_0, b_1 \dots$ are const. & m, n +ve integers is called rational algebric fraction

34) The fractions in which the deno. are linear or quadratic fu's of x & numerators are of lower degree than denominator called partial fraction

35) To every non-repeated linear factor $(x-a)$ in denominator gives a partial fraction of form $\frac{A}{x-a}$ or $\frac{\text{const.}}{x-a}$

36> If linear factor is repeated r times i.e. $(x-b)^r$, corresponds

$$\frac{B_1}{x-b} + \frac{B_2}{(x-b)^2} + \frac{B_3}{(x-b)^3} + \cdots + \frac{B_r}{(x-b)^r}$$

37> Non-repeated quadratic factor x^2+px+q gives partial fraction of the form $\frac{Cx+D}{x^2+px+q}$

38> If quadratic factor is repeated s times i.e. $(x^2+kx+l)^s$ then

$$\frac{E_1x+F_1}{x^2+kx+l} + \frac{E_2x+F_2}{(x^2+kx+l)^2} + \cdots + \frac{E_sx+F_s}{(x^2+kx+l)^s}.$$

39> If the given fraction is $f(x)/\phi(x)$ & $\phi(x) = (x-a)^r \cdot \psi(x)$, For partial fraction we put $x-a=y$

40> $$\int f[x, (ax+b)^{1/n}]\,dx = \int f\left(\frac{t^n-b}{a}, t\right)\frac{nt^{n-1}}{a}\,dt$$

41> Rational fun^n $(ax+b)^{1/n}$ & x can be evaluated by substitution $t^n = ax+b$ OR $(ax+b)^{1/n} = t$

42> For $\int \frac{x}{(x-3)\sqrt{x+1}}$ put $\sqrt{x+1} = t$

43) $\int x^m (a+bx^n)^p dx =$
$$\frac{x^{m-n+1}(a+bx^n)^{p+1}}{b(np+m+1)} - \frac{a(m-n+1)}{b(np+m+1)}\int x^{m-n}(a+bx^n)^p dx$$

44) $\int x^m (a+bx^n)^p dx$
$$= \frac{x^{m+1}(a+bx^n)^p}{a(m+1)} - \frac{\{np+m+n+1\}b}{a(m+1)}\int x^{m+n}(a+bx^n)^p dx$$

45) $\int x^m (a+bx^n)^p dx$
$$= \frac{x^{m+1}(a+bx^n)^p}{np+m+1} + \frac{anp}{np+m+1}\int x^m (a+bx^n)^{p-1} dx$$

46) $\int x^m (a+bx^n)^p dx$
$$= -\frac{x^{m+1}(a+bx^n)^{p+1}}{an(p+1)} + \frac{np+n+m+1}{an(p+1)}\int x^m (a+bx^n)^{p+1} dx$$

47) In $\int x^m (a+bx^n)^p dx$

if p is +ve integer then $(a+bx^n)^p$ can be expanded by Bionomial theo into finite series.

48) In $\int x^m (a+bx^n)^p dx$
if $(m+1)/n$ is an integer then put $\frac{m+1}{n} = j+1$ so give integral

$$\int x^{n-1} (x^n)^j (a+bx^n)^p dx$$

49> In $\int x^m (a+bx^n)^p dx$
if $p + \frac{(m+1)}{n}$ an integer, p

not integer, in this case
we put $x = \frac{1}{t}$ & integral becomes

$-\int \frac{1}{t^{m+2}} \left(a+\frac{b}{t^n}\right)^p dt$ OR

$-\int t^{-(m+np+2)} (b+at^n)^p dt$

50> $\int x^m (a+bx^n)^p dx$ can be connected
with any of one
1) $\int x^{m-n} (a+bx^n)^p dx$ 2) $\int x^m (a+bx^n)^{p-1} dx$
3) $\int x^{m+n} (a+bx^n)^p dx$ 4) $\int x^m (a+bx^n)^{p+1} dx$
5) $\int x^{m-n} (a+bx^n)^{p+1} dx$ 6) $\int x^{m+n} (a+bx^n)^{p-1} dx$

51> $\int \sec^2 (ax+b) dx = \frac{1}{a} \tan (ax+b) dx$

52> $\int \frac{1}{x} dx = \log x$

53> $\int \cosh x = \sinh x$

54> $\int \sinh x = \cosh x$

55> $\int {}^1/_{x\sqrt{x^2-1}} dx = \sec^{-1} x$