B.SC - II yr (III sem)

Group Theory - VII

Unit - I

[16 Pages]

1) If set A has n elements & B has m elements, then product A X B has ___ elements

✓a) nm b) n-m

c) n+m d) none

2) If A & B are any sets, then ...

a) $A \subseteq A \cup B$ b) $B \subseteq A \cup B$

c) $A - B \subseteq A$ d) All are true

3) If A & B are sets then

$A \cap (B - C) =$ ___

a) $A \cap B$ ✓b) $(A \cap B) - (A \cap C)$

c) $A \cup B$ d) None

4) If the domain & co-domain of a fun are same then f is called ___

a) operator b) Transformation

✓c) both ⓐ & ⓑ d) None

5) If A & B are two sets, $f: A \rightarrow B$ is one-one, onto then $f^{-1}: B \rightarrow A$ is

a) one-one b) onto

✓c) one-one onto d) None

6) If $f: X \longrightarrow Y$ be one-one & onto mapping then ___

a) $f \circ f^{-1} = I_Y$ b) $f^{-1} \circ f = I_X$

✓c) both ⓐ & ⓑ d) None

7) Let $f : A \to B$, $g : B \to C$, & $h : C \to D$
then $(h \circ g) \circ f$ = ---
✓a) $h \circ (g \circ f)$ b) hgf
c) hfg d) None

8) A binary operation $\circ$ on a set A
is called commutative if ---
for every $a, b \in A$.
a) $a \circ b = 1$ ✓b) $a \circ b = b \circ a$
c) $a \circ b = a^{-1} \circ b^{-1}$ d) None

9) Let R be a subset of $A \times A$, &
$(a,a) \in R$ $\forall a \in R$ Then it is ---
✓a) reflexive b) Associative
c) symmetric d) None

10) Let R be a subset of $A \times A$ &
$(a,b) \in R \Rightarrow (b,a) \in R$ $\forall a,b \in R$.
Then it is ----
a) reflexive b) Antisymmetric
✓c) symmetric d) None

11) R is equivalence relation if ---
a) reflexive b) symmetric
c) transitive ✓d) all of above

12) R be equivalence relation in A
& $a, b \in R$ then -----
a) $a \in [a]$ b) $b \in [a] \Rightarrow [b] = [a]$
✓c) both a & b c) None

13) '+' is binary operation on G iff ---- $\forall a,b \in G$.

a) $a+b \in G$ b) $a-b \in G$
c) $ab \in G$ d) None

14) $(G, \cdot)$ is group then $ab \in G$, $\forall a,b \in G$ is --- property

a) closure b) Associative
c) distributive d) None

15) Group $(G,\cdot)$ is abelian if -- $\forall a,b \in G$

a) $ab = ba$ b) $a-b = ab$
c) $a/b = b/a$ d) None

16) The no. of elements in a finite group is called ---

a) degree b) order
c) index d) None

17) Set N of all natural nos. is ... w.r.t. addition

a) Group b) abelian group
c) Not a group d) None

18) Set I of all integers is .... w.r.t. addition

a) Group b) abelian group
c) Not group d) None

19) Identity element in a group is ---

a) unique b) different
c) 2 d) None

20) Inverse of each element of a group is ...

✓a) unique b) different

c) 1 d) None

21) If inverse of a is a^{-1} then $(a^{-1})^{-1}$ = ..

a) a^{-2} ✓b) a

c) a^{0} d) None

22) $(ab)^{-1}$ = ... $\forall a, b \in G$

a) $a^{-1} b^{-1}$ ✓b) $b^{-1} a^{-1}$

c) $a^{-1} b$ d) None

23) If $a, b, c \in$ Group G then $ab = ac \Rightarrow$ ....

a) $a = a$ b) $a = c$

✓c) $b = c$ d) None

24) If a & b $\in$ group G then eqns $ax = b$ & $ya = b$ have .. in G

a) 2 solns ✓b) unique solns

c) 3 solns d) None

25) An algebric structure $(G, *)$ is called semigroup if the binary operation $*$ is ... in G

✓a) associative b) distributive

c) Commutative d) None

26) ... are semigroup.

a) $(N, \cdot)$ b) $(I, +)$

c) $(R, +)$ ✓d) All

Group Theory
unit – II

1> which of the following is group w.r.t. multiplication.
a) $\{1, -1, i, -i\}$ b) $\{1, \omega, \omega^2\}$
✓c) both (a) & (b) d) None

2> we have for addition modulo m
$a +_m b = r$; ----
✓a) $0 \le r < m$ b) $a < m < r$
c) $0 > r$ d) None

3> For multiplication modulo p
$a \times_p b = r$ ----
✓a) $0 \le r < p$ b) $0 > r > p$
c) $0 < r$ d) None

4> $G = \{0, 1, 2, 3, 4, 5\}$ is finite ----
a) abelian group b) order 6
c) addition mod. 6 ✓d) All of above

5> $G = \{1, 2, 3, 4, 5, 6\}$ is finite ----
a) abelian group b) order 6
c) multiplication mod 6 ✓d) All of above

6> c is called gcd of a & b if ---
a) $c|a$, $c|b$ b) $d|a$, $d|b \Rightarrow d|c$
✓c) both (a) & (b) d) None

7> If a, b, c be three integers s.t. a & b are relatively prime & $a|bc$, then ---
✓a) $a|c$ b) $a = c$
c) $c|a$ d) None

8) p is prime integer, a, b be any two integers s.t. $p|ab$. Then ---
a) $a|p$ b) $a|ap$
✓c) $p|a$ or $p|b$ d) None

9) $13 \equiv 3 \pmod{----}$
a) 3 b) 4 ✓c) 5 d) None

10) Two permutations f & g are equal iff ---
a) same deg. b) $f(a) = g(a) \ \forall a \in S$
✓c) both ⓐ & ⓑ d) None

11) $f = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ $g = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$
then $fg =$ ---
✓a) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$
c) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ d) None

12) Inverse of cycle $(1, 2, --n)^{-1} =$
a) $(1^{-1}, 2^{-1} -- n^{-1})$ ✓b) $(n, n-1, --2, 1)$
c) $(1\ 2\ --- n)$ d) None

13) If f, g, h are cycles then $(fgh)^{-1} =$
a) (fgh) ✓b) $h^{-1}g^{-1}f^{-1}$
c) $h^{-1}f^{-1}g^{-1}$ d) None

14) Every permutation can be expressed as ---- disjoint cycles
a) addition ✓b) product
c) division d) None

15> The order of an element $a \in G$ is meant least +ve integer n, s.t. ...

✓a) $a^n = e$ b) $a^n = a$

c) $a^n = n$ d) None

16> Any non-empty subset H of a group G is called ---- of a group

✓a) Complex b) Subgroup

c) index d) None

17) A non-empty subset H of a group G is a subgroup of G iff ---

a) $a \in H, b \in H \Rightarrow ab \in H$ b) $a \in H \Rightarrow a^{-1} \in H$

✓c) both (a) & (b) d) None

18) A necessary & sufficient condition for a non-empty subset H of group G to be a subgroup is that $a, b, \in H$...

✓a) $ab^{-1} \in H$ c) $a+b \in H$

c) $a/b \in H$ d) None

19) If H is a subgroup of group G then ---

✓a) $HH = H$ b) $HH = H^2$

c) $HH = ea$ d) None

20> The order of an element of a group is same as that of ---

a) b ✓a) inverse of a

c) c d) None

21> $G = \{1, -1, i, -i\}$ then ---

a) $o(1) = 1$ b) $o(-1) = 2$

c) $o(i) = 4$ ✓d) All of above

22) which of following is cyclic permutations

a) $\begin{pmatrix} 1 & 2 & 5 & 3 & 6 & 4 \\ 2 & 4 & 5 & 1 & 6 & 3 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 2 & 3 & 5 & 4 & 6 \\ 2 & 3 & 4 & 6 & 5 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 2 & 4 & 3 & 5 & 6 \\ 2 & 1 & 3 & 4 & 5 & 6 \end{pmatrix}$ ✓c) both a & b

23) length of cycle $\begin{pmatrix} 1 & 2 & 5 & 3 & 6 & 4 \\ 2 & 4 & 5 & 1 & 6 & 3 \end{pmatrix}$ is ---

a) 5 ✓b) 4 c) 3 d) 2

24) A cycle of length 2 is called ----

a) reflexive ✓b) transposition

c) transitive d) symmetric

25) The set P_3 of all permutations of degree 3 will have ---- elements

✓a) 6 b) 5

c) 4 d) 3

26) If a & b are integers, not both 0, then they have unique gcd say c & we find int x & y s.t. --

a) $c = xa$ b) $c = xayb$

✓c) $c = xa + yb$ d) None

27) Let m be any fixed +ve integer. Then an integer is said to be congruent to another integer b mod. m if ---

a) $m | a$ b) $m | a+b$

✓c) $m | a-b$ d) None

Group Theory
Unit - III

1) H is subgroup of Group G, $a \in G$ Then $Ha = \{ha : h \in H\}$ is called ---

a) index of H
b) right coset of H
c) left coset of H
d) None

2) If H is any subgroup of G & $h \in H$ then ---

a) $Hh = H = hH$
b) $Hh = a$
c) $Hb = h^2$
d) None

3) If $a, b \in G$ & H is subgroup of G then $Ha = Hb \Leftrightarrow$ ---

a) $ab \in H$
b) $ab^{-1} \in H$
c) $a^{-1}b \in H$
d) both b & c

4) If $a, b \in G$ & H is subgroup of G then $a \in Hb \Leftrightarrow$ ---

a) $Ha = a$
b) $Ha = Hb$
c) $Hab = e$
d) None

5) Any two right cosets of a subgroup are either -----

a) disjoint
b) identical
c) a or b
d) None

6) If H is subgroup of G, then G is equal to union of all ---- of H in G.

a) right cosets
b) left cosets
c) both a or b
d) Nones

7) H is subgroup of G, then there is --- corresp. betⁿ any two right coset of H

a) 1-1 b) onto

✓c) one to one d) None

8) If H is a subgroup of a group G, the no. of distinct right cosets of H in G is called ---- H in G

a) permutation ✓b) index

c) power d) None

9) Index of H in G is denoted by.

a) [G:H] b) $i_G(H)$

c) both (a) & (b) d) None

10) H is subgroup of G. a,b∈G we say a≡b (mod H) iff ---

a) ab∈H ✓b) ab^{-1}∈H

c) a∈H d) None

11) The relation of congruency in Group G is ---

a) reflexive b) symmetric

c) Transitive d) ~~none~~ All of above

12) The order of each subgroup of a finite group is a divisor of the order of group is --- theo

a) Fermat ✓b) Lagrange

3) Euler d) None

13) If G is a finite group of order n, & a∈G, then ---

✓a) $a^n=e$ b) e=1 c) $a^n=1$ d) None

14) If n is a +ve integer & a is any integer relatively prime to n then $a^{\phi(n)} \equiv 1 \pmod{n}$, ϕ is Euler ϕ-funⁿ. is - - - theo

a) Lagrange ✓b) Euler

c) Fermat d) None

15) If H is a subgroup of a finite group G then index of H in G = - - -

✓a) $O(G)/O(H)$ b) $O(H) \cdot O(G)$

c) $O(G) + O(H)$ d) None

16) If p is prime no. & a is any integer then $a^p \equiv a \pmod{p}$ is - - - theo.

a) Lagrange b) Euler

✓c) Fermat d) None

17) H & K be finite subgroups of group G Then $O(HK)$ = - - -

✓a) $\frac{O(H)\,O(K)}{O(H \cap K)}$ b) $O(H)\,O(K)$

c) $O(H) + O(K)$ d) None

18) H & K be subgroups of a finite group G & $O(H) > \sqrt{O(G)}$, $O(K) > \sqrt{O(G)}$ Then ~~HnK =~~ - - -

a) $H \cap K = H$ ✓b) $H \cap K \neq \{e\}$

c) $H \cap K = \{\ \}$ d) $H \cap K = \{e\}$

19) Two right cosets Ha, Hb are distinct iff two left cosets - - - are distinct

✓a) $a^{-1}H$, $b^{-1}H$ b) $a^{-1}H$, bH

c) aH, bH d) aH, $b^{-1}H$

20) Every finite group G is isomorphic to a permutation group. This is - - - theo

✓a) Cayley's b) Fermat c) Euler d) None

21) The permutation group to which G is isomorphic is called ___ perm. group

a) Regular b) irregular

c) Common d) None

22) A Group G is called cyclic if a∈G, every element x∈G is of form ___

a) a b) a^2

c) a^c d) a^n, n is some integer

23) Generator of group {1, -1, i, -i} is ___

a) 1 b) i c) -i d) both (b) & (c)

24) Every cyclic group is an ___ group

a) abelian b) Normal

c) non-abelian d) None

25) If a is generator of a cyclic group G, then ___ is also generator

a) b b) a^{-1} c) e d) None

26) Every group of prime order is ___

a) abelian b) Normal

c) cyclic d) None

27) If G is an infinite cyclic group then G has exactly ___ generators

a) 1 b) 2 c) 3 d) 4

28) Every subgroup of cyclic group is ___

a) cyclic b) abelian

c) Normal d) None

Group Theory
Unit - IV

1) A subgroup H of a group G is said to be normal subgroup of G, for every $x \in G$, $h \in H$ - - -

a) $H \subseteq aH$ ✓b) $xhx^{-1} \in H$
c) $xh \in H$ d) None

2) Every group G possess atleast - - - normal subgroups.

a) 01 ✓b) 02 c) 03 d) 04

3) A group having no proper normal subgroups is called - - - group

✓a) simple b) Normal
c) abelian d) None

4) A subgroup H of a group G is normal iff $xHx^{-1} =$ - - - $\forall x \in G$

✓a) H b) G c) e d) None

5) A subgroup H of a group G is normal subgroup of G iff - - - - in G

✓a) left coset is right coset b) left coset isnot right coset
c) left coset are finite d) None

6) If product of two right cosets of H in G is again a right coset of H in G then H is - - -

✓a) Normal b) abelian
c) simple d) None

7) Intersection of any two normal subgroups of a group is - - - subgroup

✓a) Normal b) abelian
c) simple d) None

8> Every Subgroup of an abelian group is ----

a) simple ✓b) Normal
c) nonabelian d) None.

9> If a, b be two elements of a group G then b is said to be conjugate to a if $\exists\, x \in G$ s.t $b =$ ---

a) x ✓b) $x^{-1}ax$
c) $x^{-1}x$ d) None

10> The relation of conjugancy is --- relation

✓a) equivalence b) into
c) non-equivalence d) None

11> $b = x^{-1}ax$, b is called --- of a by x

✓a) Transform b) uniform
c) subgroup d) None

12> If $a \in G$, $N(a) = \{x \in G : ax = xa\}$ is ---- of a in G

✓a) Normalizer b) infinite Subgroup
c) Normal d) None

13> The normalizer $N(a)$ of $a \in G$ is --

✓a) Subgroup b) Normal
c) Transform d) None

14> a is self-conjugate iff --- $\forall x \in G$

a) a b) $x^{-1}x$
✓c) $x^{-1}ax$ d) None

15) The set Z of all self-conjugate elements of a group G is called --- of G

a) Transform ✓b) Centre

c) identity d) None

16) The centre Z of group G is --- of G

✓a) Normal subgroup b) Transform

c) non abelian subgroup d) None

17) $a \in Z$ iff ---

✓a) $N(a) = G$ b) $N(a) = N(b)$

c) $N(a) \neq G$ d) None

18) A, B are two subgroup of G, then B is said to be conjugate to A iff. $\forall x \in G$ s.t ---

a) $B = A$ ✓b) $B = x^{-1} A x$

c) $B = x^{-1} x$ d) None

19) The set of all cosets of normal subgroup is a group w.r.t. --- of complexes as composition.

a) addition ✓b) multiplication

c) substraction d) None

20) If G is group & H is normal subgroup of G then the set G/H of all cosets of H in G is group w.r.t. multiplication of cosets is called --- group

a) quotient b) factor

✓c) both (a) & (b) d) None

21) A mapping $f: G \to G'$ is homomorphism if ___ $\forall a, b \in G$

a) $f(ab) = f(a)$ ✓b) $f(ab) = f(a).f(b)$
c) $f(ab) = f(b)$ d) None

22) A homomorphism of a group into itself is called ---

a) isomorphism b) Automorphism
✓c) endomorphism d) None

23) If f is homomorphism of G into group G' then ---

a) $f(e) = e'$ b) $f(a^{-1}) = f(a)^{-1} \forall a \in G$
✓c) both (a) & (b) d) None

24) If f is homomorphism of a group G into a group G' with kernel K, then K is --- of G

✓a) Normal Subgroup b) Transform
c) Identity d) None

25) Every homomorphic image of a group G is isomorphic to some quotient group of G is --- theo. of homomorphism of groups

a) First b) Euler's
✓c) Fundamental d) None

26) An isomorphic mapping of a group G onto itself is called --- of G

✓a) Automorphism b) endomorphism
c) Homomorphism d) None