B.Sc - Third year (Sem-V)

Metric Spaces - XII

Unit - I

1) If (X,d) is metric then which of the following is true?

a) $d(x,y) \geq 0 \quad \forall x,y \in X$ b) $d(x,y)=d(y,x) \quad \forall x,y$

c) $d(x,y)=0$ iff $x=y \quad \forall x,y$ ✓d) All of above

2) In metric space symmetry property is...

a) $d(x,y)=0$ ✓b) $d(x,y)=d(y,x) \quad \forall x,y \in X$

c) $d(x,y)>0$ d) None

3) $d(x,y) \leq d(x,z)+d(z,y) \quad \forall x,y,z \in X$

(X,d) is metric space, this is --- property.

a) symmetry ✓b) triangle inequality

c) reactangle inequality d) None

4) The metric d is also called --- fun.

a) unit b) root

✓c) distance d) None

5) Let $d: R \times R \to R$, s.t. $d(x,y)=|x-y| \quad \forall x,y \in R$ is ---

a) metric b) Not metric

c) trivial metric d) None

6) X be non-empty set. The fun d defined by $d(x,y)=\begin{cases} 1 & x \neq y \\ 0 & x=y \end{cases} \quad \forall x,y \in X$

then this metric is called ---

a) trivial b) discrete

c) both (a) & (b) d) Tchebyshev

7) $d(f,g) = \sup_{x \in [0,1]} |f(x) - g(x)| \quad \forall f, g \in C[0,1]$

is ... metric

a) Tchebyshev   b) sup
c) both (a) & (b)   d) None

8) Cauchy Schwarz inequality is ...

a) $\left|\sum_{k=1}^{n} a_k b_k\right| = 1$ ✓b) $\left|\sum_{k=1}^{n} a_k b_k\right| \leq \sqrt{\sum_{k=1}^{n} a_k^2 \sum_{k=1}^{n} b_k^2}$

c) $a_k b_k = \sqrt{a_k^2}$ d) None

9) $d(x,y) = \left(\sum_{i=1}^{n} (x_i - y_i)^2\right)^{1/2} \quad \forall x = (x_1, x_2 \ldots x_n)$
$y = (y_1, y_2 \ldots y_n)$

is ... metric

a) Tchebyshev   ✓b) Euclidean
c) Trivial   d) None

10) $d'(x,y) = \sum_{i=1}^{n} |x_i - y_i|$ defined on R^n

is ... metric

a) Tchebyshev   ✓b) rectangular
c) Euclidean   d) None

11) For any two non-empty subsets A & B of metric space (X, d), distance between them is $d(A,B) = \ldots$

✓a) $\inf\{d(a,b) : a \in A,\ b \in B\}$
b) $\sup\{d(a,b) : a \in A,\ b \in B\}$
c) $\{d(a,b) : a \in A,\ b \in B\}$
d) None

12) If $A = \{x \in R : 0 < x \leq 1\}$ & d is usual metric, then $d(0, A) =$ ---

✓a) 0 b) 1 c) 2 d) 3

13) $A = \{1, \frac{1}{3}, \frac{1}{5}, \cdots \frac{1}{2n-1} \cdots\}$, $B = \{\frac{1}{2}, \frac{1}{4} \cdots \frac{1}{2n} \cdots\}$

then

$d(A, B) =$ ---

✓a) 0 b) 1 c) 2 d) 3

14) A metric d on a non-empty set X is said to be bounded if there exist real no K s.t. ---- $\forall x, y \in X$

a) $d(x,y) > K$ ✓b) $d(x,y) \leq K$

c) $d(x,y) = 0$ d) None

15) Let (X, d) be any metric space & $a \in X$ Then for $r > 0$, the set $S_r(a) = \{x \in X : d(x,a) < r\}$ is called... of radius r centred at a

a) open sphere b) open ball

✓c) both (a) or (b) d) None

16) $S_r[a] = \{x \in X : d(x,a) \leq r\}$ is called --

✓a) Closed sphere a) open sphere

c) Sphere d) None

17) For metric space (R, d) of real nos. with usual metric d, the open sphere $S_r(a)$ is ------ & closed sphere is ---

a) $(a-r, a+r)$ & $(a+r, a-r)$

✓b) $(a-r, a+r)$ & $[a-r, a+r]$

c) $(a-r, a+r)$ $(2a-r, a+r]$

d) None

18) Let (X,d) be a metric space & $S_r(x)$ the open sphere with centre x & radius r. Let A be a subset of X with diameter less than r, which intersects $S_r(x)$ then ---

a) $A = S_{2r}(x)$ b) $S_{2r}(x) \subseteq A$
✓c) $A \subseteq S_{2r}(x)$ d) None

19) Every ---- is nbd. of each of it's points.

✓a) open sphere b) closed sphere
c) $[a,b]$ d) None

20) Every set in a discrete space (X,d) is ---

✓a) open b) closed
c) half open half closed d) None

21) On the real line with usual metric the singleton set $\{x\}$ is ---

a) open ✓b) not open
c) closed d) None

22) Every open set in (X,d) is open in (X,d'), then d, d' are called ---

✓a) equivalent b) congruent
c) symmetric d) None

23) --- open set

a) Φ b) X
c) open sphere ✓d) ~~All~~ All of above

M.S - Unit - I

24) In any metric space (X,d), the union of ~~open~~ an arbitrary family of open sets is - - -
✓a) open b) closed
c) not open d) None

25) In any metric space (X,d) the intersection of finite no. of open sets is - - -
✓a) open b) closed
c) not open d) None

26) limit point is also called - - - point
a) Cluster b) condensation
c) accumulation ✓d) All of above

27) The set of all limit points of A is called - - - of A
✓a) derived point b) isolated point
c) adherent point d) None

28) A subset F of a metric space (X,d) is said to be closed if F contains all it's - - - point
✓a) limit b) Adherent
c) isolated d) None

29) Let (X,d) be any metric space. A subset F of X is closed iff it's complement is - - -
a) closed ✓b) open
c) not open d) None

30) Every closed sphere is a ___ set.
a) open b) closed
c) not open d) None

31) In any M.S (X,d) intersection of an arbitrary family of closed set is ___
a) open b) closed
c) not open d) None

32) Let (X,d) be M.S. & $Y \subseteq X$, then a subset A of Y is open in (Y, d_Y) iff $\exists$ set G open in (X,d) s.t.
a) $A = G \cup Y$ b) $A = G \cap Y$
c) $A = G$ d) None

33) A be subset of M.S. (X,d) then $\overline{A} =$ ___
a) $A \cup A'$ b) $A \cap A'$
c) $2A$ d) None

34) A & B are subsets of (X,d), then which is true?
a) $\overline{A}$ is closed b) $A \subseteq B \Rightarrow \overline{A} \subseteq \overline{B}$
c) $\overline{A \cup B} = \overline{A} \cup \overline{B}$ d) All are true

35) $A, B \subseteq X$, then ___
a) $\overline{A \cap B} = \overline{A} \cap \overline{B}$ b) $A = \overline{A}$, iff A is closed
c) $\overline{A}$ is smallest closed superset of A
d) ~~Non~~ All are true

Metric spaces - XII

Unit - II

1) The seq^n $\{a_n\}$ of points of X is said to converge to pt. a if $\epsilon > 0$, $\exists$ m s.t.

a) $d(a_n, a) < \epsilon \; \forall n \geq m$ b) $d(a_n, a) \to 0$ as $n \to \infty$

✓c) both (a) & (b) d) None

2) $\{a_n\}$ is said to be Cauchy seq^n if $\epsilon > 0$, $\exists$ +ve n_0 s.t. _

a) $d(x_n, x_m) < \epsilon \; \forall n, m \geq n_0$ b) $d(x_n, x_m) \to 0$, $n, m \to \infty$

✓c) both (a) & (b) d) None

3) Every ___ seq^n is Cauchy seq^n

a) divergent ✓b) convergent

c) oscillating d) None

4) A metric space (X, d) is said to be complete if every Cauchy seq^n ___

a) diverges b) converges

✓c) converges to pt of X d) None

5) Let (X, d) be any M.S. & A be any non-empty subset of X, then $x \in \bar{A}$ iff there exists a seq^n $\{x_n\}$ in A s.t. ___

✓a) $x_n \to x$, $n \to \infty$ b) $x_n \to 0$

c) $x_n \to 1$ d) None

6) (X, d) be complete M.S. Y be subspace of X then Y is complete iff ___ in (X, d)

✓a) Y is closed b) Y is open

c) Y is not open d) None

7) Let (X,d) be c.m.s. & let $\{F_n\}$ be a decreasing seq^n of non-empty closed subsets of X s.t. $d(F_n) \to 0$ as $n \to \infty$. Then $F = \bigcap_{n=1}^{\infty} F_n$ contains exactly one pt. is ... theo

a) Baire's ✓b) Cantor's intersection

c) First category d) None

8) If $\{A_n\}$ is a seq^n of nowhere dense sets in a c.m.s (X,d) then $X \neq \bigcup_{n=1}^{\infty} A_n$ is ... theo

✓a) Baire's b) First category

c) Second category d) None

9) Every complete m.s. is of ... category

a) First b) second

c) Third d) None

10) A fu^n $f: X \to Y$ is conti. iff ...

a) $f(S_\delta(a) \subseteq f(a)$ b) $f(a) \subseteq S_\delta(a)$

✓c) $f(S_\delta(a)) \subseteq S_\epsilon(f(a))$ d) None

11) If (X,d) is discrete m.s. then $f: X \to Y$ is ... on X

✓a) continuous b) discontinuos

c) cannot say d) None

12) (X,d_1) & (Y,d_2) be 2 m.s. & $f: X \to Y$ Then f is conti. at $a \in X$ iff $a_n \to a \Rightarrow$ ...

✓a) $f(a_n) \to f(a)$ b) $f(a_n) = 0$

c) $f(a_n) \neq f(a)$ d) None

13) Let (X,d_1) & (Y,d_2) be two metric spaces, then $f: X \to Y$ is conti. iff $f^{-1}(G)$ is open in X, whenever ---

a) G open in X ✓b) G open in Y

c) G closed in Y d) None

14) Let (X,d_1) & (Y,d_2) be metric spaces show that $f: X \to Y$ is conti. iff --- $A \subseteq X$

a) $f(\bar{A}) = \overline{f(A)}$ ✓b) $f(\bar{A}) \subseteq \overline{f(A)}$

c) $f(\bar{A}) \supseteq \overline{f(A)}$ d) None

15) A funⁿ $f: X \to Y$ is said to be a homeomorphism if ---

a) f is 1-1, onto b) f is conti.

c) f^{-1} is conti. ✓d) All of above

16) A function $f: X \to Y$ is called isometry if $d(x,y) =$ --- $\forall x, y \in X$

✓a) $d'(f(x), f(y))$ b) 1

c) $d(x,y)$ d) None

17) Image of a cauchy seqⁿ under a uniformly Conti. fun is --- seqⁿ

✓a) cauchy b) divergent

c) oscillating d) None

18) A point $x \in X$ is called fixed point of $f: X \to Y$ if ---

a) $f(x) = 1$ ✓b) $f(x) = x$

c) $f(x) = 0$ d) None

19) Any contraction mapping f of a non-empty complete metric space (X,d) into itself has unique fixed point is .. theo

a) Cantor's ✓b) Banach fixed pt.
c) Baire's d) None.

20) A mapping $f: X \longrightarrow X$ is said to be a contraction mapping if $\exists$ +ve $\alpha<1$ s.t. ~~de~~ __

✓a) $d(f(x), f(y)) \leq \alpha d(x,y) \;\; \forall x,y \in X$
b) $d(f(x), f(y)) > d(x,y) \;\; \forall x,y \in X$
c) $\alpha\, d(f(x), f(y)) < d(x,y) \;\; \forall x,y \in X$
d) None

21) $f(x) = x^2$ $\quad x \in [0, \frac{1}{3}]$ then $\alpha =$ __

a) 1/3 ✓b) 2/3
c) 4/3 d) 5/3

22) Let (X, d_1) & (Y, d_2) be two M.S. A fun $f: X \longrightarrow Y$ is said to be uniformly conti. if for each $\epsilon > 0$ $\exists$ $\delta > 0$ s.t. ___ $\forall x,y \in X$

✓a) $d_2(f(x), f(y)) < \epsilon$ whenever $d_1(x,y) < \delta$
b) $d_1(f(x), f(y)) < \epsilon$ whenever $d_2(x,y) < \delta$
c) $d_2(f(x), f(x)) \leq \epsilon$ whenever $d_2(x,y) \leq \delta$
d) None.

B.Sc. - Third year (Sem-V)
Metric spaces
Unit - III

1) A subset A of a metric space (X, d) is said to be compact if every open cover of A admits ......
a) finite subcover ✓ b) subcovers
c) infinite subcover d) None

2) Any closed interval with usual metric is ...
a) compact ✓ b) Not-compact
c) seprated d) None

3) Every closed subset of a compact metric space is ...
a) connected b) compact ✓
c) not compact d) None

4) Every compact subset F of a m.s. (X, d) is ....
a) open b) closed ✓
c) half open d) None

5) A subset A of a compact metric space (X, d) is itself compact iff it is ~~closed~~ ---- in (X, d)
a) closed ✓ b) open
c) half open d) None

6) Every closed & bounded subset of the real line is compact. is ... theo
a) Heine-Borel ✓ b) Banach
c) Cauchy d) None

7) continuous image of a compact set is.....

a) discontinuous ✓b) compact

c) Non-compact d) None

8) Let A be a non-empty compact subset of a m.s. (X, d) & let F be a closed subset of X s.t. $A \cap F = \phi$ then ---

a) $d(A,F)=0$ b) $d(A,F)<0$

✓c) $d(A,F)>0$ d) None

9) A family of subset of a non-empty set X is said to have FIP if every finite subfamily has .... intersection

a) empty ✓b) non-empty

c) {0} d) None

10) A metric space (X,d) is said to have ----- property if every infinite subset of X has a limit point.

✓a) Bolzano-weierstrass b) cauchy

c) Heine-Borel d) None

11) A metric space (X,d) is sequentially compact if every sequence $\{x_n\}$ in X has ...

a) Subsequence ✓b) Convergent subseq^n.

c) divergent subseq^n d) None

12) A m.s (X,d) is sequentially compact iff it has ----- property

✓a) Bolzano-weierst. b) cauchy

c) Heine-Borel d) None

13) Every compact M.S. (x,d) is - - -
a) Connected ✓b) seqⁿ. compact
c) not. seqⁿ. compact d) None

14) A subset A of M.S. (x,d) is relatively compact if - - - is compact.
✓a) $\overline{A}$ b) $4A^2$ c) $3A^3$ d) None

15) Every totally bounded M.S. (x,d) is . . .
✓a) seprable b) non-seprable
c) cannot say d) None

16) Every - - - M.S. is seprable
a) compact b) totally bounded
✓c) both (a) & (b) d) None

17) A M.S. (x,d) is totally bounded iff every seqⁿ in X contains - - - - -
✓a) cauchy subseqⁿ b) subseqⁿ
c) divergent subseqⁿ d) None

18) A M.S. (x,d) is sequentially compact iff it is - - -
a) complete b) totally bounded
✓c) both (a) & (b) d) None

19) A subset A of M.S. (x,d) is totally bounded iff - - - -
✓a) $\overline{A}$ is totally bounded
b) $\overline{A}$ is seprable
c) $\overline{A}$ bounded above
d) None

20) Every open cover of a sequentially compact M.S. (X,d) has Lebesgue no. is ....

a) Lebesgue covering lemma b) Banach theo.
c) Heine Borel theo. d) None

21) A M.S. is compact iff it is ....
a) complete b) totally bounded
✓c) both (a) & (b) d) None

22) A closed subspace of a complete M.S. is compact iff it is ....
✓a) totally bounded b) bounded
c) seprable d) None

23) Two sets A & B are seprated if ____
a) $A \cap \overline{B} = \phi$ b) $\overline{A} \cap B = \phi$
✓c) (a) & (b) both d) None

24) If $A = \{x : -\infty < x < 0\}$ $B = \{x : 0 \leq x < \infty\}$ then A & B are ___
a) disjoint b) not seprated
✓c) (a) & (b) d) None

25) A subset A of a M.S. (X,d) is said to be connected if ....
✓a) it cannot be expressed as union of two non-empty seprated set
b) it can be expressed as union of two non-empty seprated set
c) it can be expressed as intersection of two ~~non~~ non-empty seprated set
d) None

26) Let Y be a subset of M.S. (X,d) then - -

a) Y is connected.

b) Y cannot be expressed as disjoint union of 2 non-empty closed sets in Y.

c) ϕ & Y are only two sets which are both open & closed in Y

✓d) All of above

27) A subset Y of M.S. X is disconnected iff $Y \subseteq G_1 \cup G_2$, where G_1 & G_2 are open in X s.t. - - -

✓a) $Y \cap G_1 \neq \phi$ $Y \cap G_2 \neq \phi$ $G_1 \cap G_2 \cap Y = \phi$

b) $Y \cap G_1 = \phi$ $Y \cap G_2 \neq \phi$

c) $Y \cap G_1 = \phi$ $G_1 \cap G_2 \cap Y \neq \phi$

d) None

28) Continuous image of connected set is - - -

a) Seprated b) disconnected.

✓c) connected d) None

29) The union of two connected sets, having non-empty intersection is - - -

✓a) connected b) disconnected

c) open & closed d) None

30) A be connected subset of M.S. X, & $B \subseteq X$ s.t. $A \subseteq B \subseteq \bar{A}$ then - B is - - -

a) bounded ✓b) connected

c) Seprated d) None.