B.Sc - III yr (VI sem)

Integral Transform - XVI

19 pages

Unit - I

1. Laplace transform of $f(t)$ is ---

✓ a) $F(s) = \int_0^\infty e^{-st} f(t)\, dt$

b) $F(s) = e^{-st}$ c) $F(s) = e^{-st} f(t)$

d) None

2. $L(1)$ = ---

✓ a) $1/s$ b) $2/s$ c) $3/s$ d) None

3. $L(t^n)$ = --- $n = 0, 1, 2, ---$

a) n ✓ b) $\frac{n!}{s^{n+1}}$

c) s^n d) None

4. $L(e^{at})$ = ---

a) $s-a$ b) $s+a$

✓ c) $1/s-a$ d) None

5) $L(e^{-at})$ = ---

a) s ✓ b) $1/s+a$

c) 0 d) None

6) $L(\cosh at)$ = ---

✓ a) $\frac{s}{s^2-a^2}$ b) $\frac{s}{s^2+a^2}$

c) s d) None

7. $L(\sinh at) =$ ___

a) $\dfrac{a}{s^2-a^2}$ b) $\dfrac{1}{s^2-a^2}$

c) as d) None

8. $L(\sin at) =$ ___

a) a b) $\dfrac{a}{s^2+a^2}$

c) s^2-a^2 d) None

9. $L(\cos at) =$ ___

a) a^2 b) s/s^2+a^2

c) a^2+s^2 d) None

10. $L(\cos 2t) =$ ___

a) 4 b) s/s^2+4

c) a^2+s^2 d) None

11. $L[a f_1(t) + b f_2(t)] = \underline{a L[f_1(t)] + b L[f_2(t)]}$

12. $L[e^{at} f(t)] = F(s-a)$ is ___ shifting theo.

a) First b) Second

c) Third d) None

13. $L(e^{at} t^n) = \underline{\dfrac{n!}{(s-a)^{n-1}}}$

14) $L(e^{at} \cosh bt) = \dfrac{s-a}{(s-a)^2-b^2}$

15) $L(e^{at} \sinh bt) = \dfrac{b}{(s-a)^2-b^2}$

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16. $L(e^{at}\sin bt) = \dfrac{b}{(s-a)^2+b^2}$

17. $L(e^{at}\cos bt) = \dfrac{s-a}{(s-a)^2+b^2}$

18. $L[f'(t)] =$ ----- (L.T. of derivative)

a) $s\,L[f(t)]$ ✓b) $s\,L[f(t)] - f(0)$

c) $L[f(t)$ d) None

19. $L[f^n(t)] =$

$s^n L[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) \cdots f^{n-1}$

20. L.T. of Integral $L\left[\int_0^t f(t)\,dt\right] =$ ---

a) $\frac{1}{s}$ ✓b) $\frac{1}{s}F(s)$

c) $F(s)$ d) None

21) L.T. of $t.f(t)$:

$L[t^n.f(t)] =$ ---

✓a) $(-1)^n.\dfrac{d^n}{ds^n}F(s)$ b) $(-1)^n$

c) $F(s)$ d) None

22) L.T. of $\frac{1}{t}\cdot f(t)$:

$L\left[\frac{1}{t}f(t)\right] =$ -------

a) $F(s)$ ✓b) $\int_s^\infty F(s)\,ds$

c) $f'(t)$ d) None

23. $L\left[\frac{\sin 2t}{t}\right] = $ ___

a) $\cot^{-1}\frac{s}{2}$ b) $\frac{\pi}{2} - \tan^{-1}\frac{s}{2}$

✓ c) both ⓐ & ⓑ d) None

24. $L\left[\int_0^t \frac{\sin t}{t}\, dt\right] = $ ___

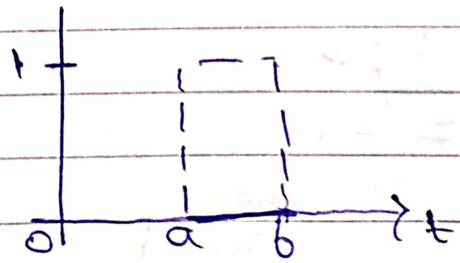
a) $\frac{1}{s}$ ✓ b) $\frac{1}{s}\left[\frac{\pi}{2} - \tan^{-1}s\right]$

c) $\frac{\pi}{2}$ d) None

25. unit step fu^n is defined by

$$u(t-a) = \begin{cases} 0 & t < a \\ 1 & t \geq a \end{cases}$$

26. Graph of $u(t-a) - u(t-b)$ is



27. L.T. of unit step fu^n

$L[u(t-a)] = $ ___

a) e^{as} ✓ b) $\frac{e^{-as}}{s}$

c) $1/s$ d) None

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28. $L[f(t-a).u(t-a)] = e^{-as}F(s)$ is __ shifting theo.

a) First   b) second

c) Third   d) None

29. $L[f(t).u(t-a)] = ---$

a) $e^{-as} L[f(t+a)]$ b) e^{-ay}

c) e^{ay} d) None

30. unit impulse fun. is defined as

$$\delta(t-a) = \begin{cases} \infty & t=a \\ 0 & t\neq a \end{cases}$$

31. $\int_0^\infty \delta(t-a)\, dt = \underline{1}$

32. L.T. of unit impulse fun is

$$\int_0^\infty f(t).\delta(t-a)\,dt = \underline{f(\eta)} \quad a<\eta<a+\epsilon$$

33. f(t) is periodic fun with period T. Then

$$L[f(t)] = \frac{\int_0^T e^{-st}.f(t)\,dt}{1-e^{-sT}}$$

34. $\int_0^\infty f(t).\delta(t-a)\,dt = \underline{f(a)}$

35. $\int_{-\infty}^{\infty} e^{-st}\delta(t-2) = \underline{e^{-10}}$

36. $\int_{-\infty}^{a} f(t).\ \delta'(t-a)dt = -f'(a)$

37. $L(e^{+4t}) =$ ___

✓a) $\frac{1}{s-4}$ b) $\frac{1}{s-2}$ c) $\frac{1}{s+4}$ d) None

38. $L(\cos 2t) =$ ___

a) $\frac{s}{s^2+2}$ ✓b) $\frac{s}{s^2+4}$

c) $\frac{s}{s^2+3}$ d) None

39. $L[e^{3t} f(t)] =$ ___

✓a) $F(s-3)$ b) $F(s-9)$

c) $F(s)$ d) None

40. $L[t^2] =$ ___

✓a) $\frac{2!}{s^3}$ b) $\frac{4}{s}$

c) $\frac{s^2}{4}$ d) None

41. $L[t^{-1/2}] =$ ___

✓a) $\sqrt{\pi}$ b) π c) 0 d) None

42. $\Gamma(1/2) =$ ___

✓a) $\sqrt{\pi}$ b) π c) 0 d) None

43. $L\left[\int_0^t f_1(x).\, f_2(t-x)\, dx\right]$ $\begin{cases} Lf_1(t) = F_1(s) \\ Lf_2(t) = F_2(s) \end{cases}$

$= F_1(S).\, F_2(s)$

is called convolution theo.

or

$F_1(S) : F_2(S) = L^{-1} \int_0^t f(x).\, f_2(t-x)\, dx$

44. $L(t.\sinh at) = \dfrac{2as}{(s^2-a^2)^2}$

45) $L(t^2.\cos at) = \dfrac{2s(s^2-3a^3)}{(s^2+a^2)^3}$

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IIIyr (sem-VI)
Integral Transform - XVI

Unit - II (Inverse Lap. Trans.)

1. Inverse Laplace transform
If $L[f(t)] = F(s)$ then
$L^{-1}[F(s)] = f(t)$
L^{-1} is called inverse L.T.

2. $L^{-1}\left(\frac{1}{s}\right) = 1$

3. $L^{-1}\left(\frac{1}{s^n}\right) = \frac{t^{n-1}}{(n-1)!}$

4. $L^{-1}\left(\frac{1}{s-a}\right) = e^{at}$

5. $L^{-1}\left(\frac{s}{s^2-a^2}\right) = \_\_\_$

a) sin at b) cos at ✓c) cosh at d) None

6. $L^{-1}\left(\frac{1}{s^2-a^2}\right) = \_\_\_$

✓a) $\frac{1}{a}$ sinh at b) cos at c) sin at d) None

7. $L^{-1}\left(\frac{1}{s^2+a^2}\right) = \_\_\_$

✓a) $\frac{1}{a}$ sin at b) cos at

c) sin at d) None

8. $L^{-1}(F(S-a)) = \_\_\_\_$

a) $e^{-at} f(t)$ b) $e^{at} f(t)$

c) $f(t)$ d) None

9. $L^{-1}\left[\frac{1}{(S-a)^2+b^2}\right] = \_\_\_\_$

a) $\frac{1}{b} e^{at}$ b) $\frac{1}{b} e^{at} \sin bt$

c) $\frac{1}{b}$ d) None

10) $L^{-1}\left[\frac{S-a}{(S-a)^2+b^2}\right] = \_\_\_\_$

a) e^{at} b) $e^{at} \cos bt$

c) e^{-at} d) None

11. $L^{-1}\left[\frac{1}{(S-a)^2-b^2}\right] = \_\_\_\_$

a) $\frac{1}{b} e^{bt}$ b) $\frac{1}{b} e^{at} \sinh bt$

c) ab. d) None

12. $L^{-1}\left[\frac{S-a}{(S-a)^2-b^2}\right] = \_\_\_\_$

a) e^{at} b) $e^{at} \cosh bt$

c) $\sin bt$ d) None

13. $L^{-1}\left[\frac{1}{S-2}\right] = \_\_\_\_$

a) e^{-2t} b) e^{2t} c) e^{3t} d) e^{4t}

14. $L^{-1}\left[\frac{1}{s^2-9}\right] = $ ------

a) $\sinh 3t$ b) $\frac{1}{3}\sinh 3t$

c) $\cos 3t$ d) None

15. $L^{-1}\left[\frac{s}{s^2-16}\right] = $ ------

a) $\cosh 4t$ b) $\sinh 4t$

c) $\sin 4t$ d) None

16. $L^{-1}\left[\frac{1}{s^2+25}\right] = $ ------

a) $\frac{1}{5}\sin 5t$ b) $\sin 5t$

c) $\cos 5t$ d) None

17. $L^{-1}\left[\frac{s}{s^2+9}\right] = $ ------

a) $\sin 3t$ b) $\cos 3t$

c) e^{at} d) None

18. $L^{-1}\left[\frac{1}{(s-2)^2+1}\right] = $ ------

a) $\sin 3t$ b) $e^{2t}.\sin t$

c) e^{2t} d) None

19. $L^{-1}\left[\frac{s-1}{(s-1)^2+4}\right] = e^{t}.\cos 2t$

20. $L^{-1}\left[\frac{1}{(S+3)^2-4}\right] =$ ______

a) $\frac{1}{2}e^{-3t}\sinh 2t$ b) e^{-3t}

c) $\sinh 2t$ d) None

21. $L^{-1}\left[\frac{S+2}{(S+2)^2-25}\right] =$ $\underline{e^{-2t}\cosh 5t}$

22. $L^{-1}\left[\frac{1}{2S-7}\right] =$ ______

a) e^{2t} b) $\frac{1}{2}e^{\frac{7}{2}t}$

c) e^{at} d) None

23. multiplication by s :

$L^{-1}[S \cdot F(S)] = \underline{\frac{d}{dt}f(t) + f(0)\,\delta(t)}$

24. $L^{-1}\left[\frac{S}{S^2+1}\right] =$

a) $\sin t$ b) $\cos t$

c) $-\sin t$ d) None

25) $L^{-1}\left[\frac{S}{4S^2-25}\right] =$ ______

a) $\cos 5t$ b) $\frac{1}{4}\cosh\frac{5}{2}t$

c) $\sin 5t$ d) None

26. Division by S

$$L^{-1}\left[\frac{F(s)}{s}\right] = \int_0^t L^{-1}[F(s)]\,ds = \int_0^t f(t)\,dt$$

27. $L^{-1}\left[\frac{1}{s(s+a)}\right] = $ ---

✓ a) $\frac{1}{a}[1-e^{-at}]$ b) e^{at}

c) e^{-at} d) None

28. First shifting property:

If $L^{-1}[F(s)] = f(t)$ then

$$L^{-1}[F(s+a)] = e^{-at} L^{-1}[F(s)]$$

29. second shifting property:

$$L^{-1}[e^{-as}F(s)] = f(t-a).\,u(t-a)$$

30. Inv. L.T. of derivative.

$$L^{-1}\left[\frac{d}{ds}F(s)\right] = -t\,L^{-1}[F(s)] = -t\,f(t)$$

or

$$L^{-1}[F(s)] = -\frac{1}{t}\,L^{-1}\left[\frac{d}{ds}F(s)\right]$$

31. $L^{-1}\left[\tan^{-1}\frac{1}{s}\right] = $ ---

a) sint b) cost

✓ c) $\frac{\sin t}{t}$ d) None

32. Inverse L.T. of Integrals:

$$L^{-1}\left[\int_s^{\infty} F(s)\,ds\right] = \frac{f(t)}{t} = \frac{1}{t}\cdot L^{-1}[F(s)]$$

or

$$L^{-1}[F(s)] = t \cdot L^{-1}\left[\int_s^{\infty} F(s)\,ds\right]$$

33. Inverse L.T. by Convolution:

$$L^{-1}[F_1(s) \cdot F_2(s)] = \int_0^t f_1(x) \cdot f_2(t-x)\,dx$$

34. $L[y''] = s^2\bar{y} - sy(0) - y'(0)$

35. $L[y'] = s\bar{y} - y(0)$

36. $L^{-1}\left[\log \frac{s^2-1}{s^2}\right] = \_\_\_$

a) 2 b) $\frac{2}{t}(1-\cosh t)$

c) $\sinh t$ d) None

37. $L^{-1}[\cot^{-1}(1+s)] = \_\_\_$

a) t b) $\frac{1}{t}e^{-t}\sin t$

c) t^2 d) None

38. $L^{-1}\left[\frac{1}{(s+2)^2}\right] = \_\_\_$

a) $t^4/4!$ b) t

c) $3t$ d) None

III yr (sem - VI)
Integral Transform - XVI

Unit - III (Integral Transform)

1. The integral transform $F(S)$ of a fun $f(x)$ with Kernel $K(S, x)$ is defined as.

$$I[f(x)] = F(S) = \int_a^b f(x).K(S, x)\,dx$$

2. Fourier complex transform with Kernel $K(S, x) = e^{-isx}$ is.

$$F(f(x)) = F(S) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\,dx$$

3. Laplace transform with Kernel $K(S, x) = e^{-Sx}$ is --

$$L[f(x)] = F(S) = \int_0^{\infty} f(x).\, e^{-Sx}\,dx$$

4. Fourier Sine transform with Kernel $K(S, x) = \sin Sx$

$$F_s[f(x)] = F(S) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin Sx\,dx$$

5. Fourier cosine transform with kernel $K(s,x) = \cos sx$

$$F_c[f(x)] = F(s) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos sx \, dx$$

6. Fourier cosine transform kernel $K(s,x) =$ ___

a) $\sin sx$ b) $\cos sx$

c) $\tan sx$ d) None

7. Fourier sine transform kernel $K(s,x) =$ ___

a) $\sin sx$ b) $\cos sx$

c) $\tan sx$ d) None

8. Hankel transform :

$$H[f(x)] = F(s) = \int_0^{\infty} f(x) \cdot x J_n(sx) \, dx$$

Here kernel $K(s,x) = x J_n(sx)$

9. Hilbert transform :

$$F(s) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(x)}{s-x} dx$$

Here kernel $= \frac{1}{s-x}$

10. Mellin transform:

$$M[f(x)] = F(s) = \int_0^{\infty} f(x) \cdot x^{s-1} \cdot dx$$

Here Kernel $K(s, x) = x^{s-1}$.

11. Fourier integral of f(x) is $f(x) = \_\_\_$

a) 0

✓ b) $\frac{1}{\pi} \int_0^{\infty} \int_{-\infty}^{\infty} f(t) \cos u(t-x) \, du \, dt$

c) $-\pi$

d) None

12. Fourier sine transform of $f(x) = \_\_\_$

✓ a) $\frac{2}{\pi} \int_0^{\infty} \int_0^{\infty} f(t) \sin ut . \sin ux \, du \, dt$

b) $\frac{2}{\pi} \int_0^{\infty} \sin ut \, dt$

c) 0

d) None

13. Fourier cosine transform of $f(x) = \_\_$

✓ a) $\frac{2}{\pi} \int_0^{\infty} \int_0^{\infty} f(t) \cos ut . \cos ux \, du \, dt$

b) $\frac{2}{\pi} \int_0^{\infty} \int_0^{\infty} f(t) \, dt$

c) 0

d) None

14) If $f(x)$ is odd then ___

a) $\int_{-a}^{a} f(x)dx = 0$ b) $\int_{-a}^{a} f(x)dx = 1$

c) $\int_{-a}^{a} f(x)dx = 2$ d) None

15. If $f(x)$ is even then ___

a) $\int_{-a}^{a} f(x)dx = 0$ b) $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

c) $\int_{-a}^{a} f(x) = 2$ d) None

16. $f(t)$ is odd then $f(t)\cos ut$ is <u>odd</u> & <u>$f(t)\sin ut$ is even</u>.

17. $f(t)$ is even then $f(t)\sin ut$ is <u>odd</u> & <u>$f(t)\cos ut$ is even</u>

18. $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-iux} du \int_{-\infty}^{\infty} f(t) e^{iut} dt$

is ___

a) sine Integral
b) cosine Integral
c) Fourier complex Integral
d) None

19. $f(x) = \frac{2}{\pi}\int_{0}^{\infty} \cos ux \, du \int_{0}^{\infty} f(t)\cos ut \, dt$

is <u>Fourier cosine integral.</u>

Properties of Fourier transform.

20 **Linear property:**

If $F_1(s)$ & $F_2(s)$ are fourier transform of $f_1(x)$ & $f_2(x)$ resp. then

$$F[a f_1(x) + b f_2(x)] = a F_1(s) + b F_2(s)$$

21. **Change of scale property:**

If $F(s)$ is complex fourier transform of $f(x)$ then

$$F\{f(ax)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$$

22. **Shifting property:**

If $F(s)$ is complex fourier transform of $f(x)$, then

$$F[f(x-a)] = e^{isa} F(s)$$

23. **Modulation theorem:**

If $F(s)$ is complex fourier transform of $f(x)$ then

$$F[f(x)\cos ax] = \frac{1}{2}[F(s+a) + F(s-a)]$$

24. If $F[f(x)] = F(s)$, then $F[x^n f(x)] =$ ---

a) $-\frac{d^n}{ds^n} F(s)$ b) $(-i)^n \cdot \frac{d^n}{ds^n} F(s)$

c) $-\frac{d}{ds} F(s)$ d) None

25. $F_s[af(x)+bg(x)] = aF_s[f(x)]+bF_s[g(x)]$

26) $F_s[f(ax)] = \frac{1}{a} F_s\left(\frac{s}{a}\right)$

27. $F_c[f(ax)] = \frac{1}{a} F_c\left(\frac{s}{a}\right)$

28. $F_s\{f(x)\sin ax\} = \frac{1}{2}\left[F_c(s-a) - F_c(s+a)\right]$

29. $F_c\{f(x)\sin ax\} = \frac{1}{2}\left[F_s(s+a) - F_s(s-a)\right]$

30. $F_s\{f(x).\cos ax\} = \frac{1}{2}\left[F_s(s+a) + F_s(s-a)\right]$

* $F_s(s)$ & $F_c(s)$ are fourier sine & cosine transforms of $f(x)$ resp.

31. Fourier sine transform of $\frac{1}{x}$ is ---

a) $\sqrt{\pi}$ b) π ✓c) $\sqrt{\pi/2}$ d) 0

32. Fourier sine transform of e^{-ax} is ---

a) 0 b) π ✓c) $\sqrt{\frac{2}{\pi}} \frac{s}{a^2+s^2}$ d) None

33. Fourier cosine transform of e^{-ax} is ---

a) 0 b) π c) $\sqrt{\frac{2}{\pi}} \frac{a}{a^2+s^2}$ d) None