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NEPSA—3012—2025

FACULTY OF SCIENCE & TECHNOLOGY

B.Sc. (First Year) (Second Semester) EXAMINATION

APRIL/MAY, 2025

(NEP 2020 Pattern)

MATHEMATICS

Paper SMATCT1151

(Analytical Geometry)

(Thursday, 17-4-2025) **Time : 10.00 a.m. to 12.00 Noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are carry equal marks.

(*ii*) Question no. **1** is compulsory.

(*iii*) Solve any *three* of the remaining *five* questions (Q. No. **2** to Q. No. **6**).

(*iv*) Figures to the right indicate full marks.

1. Solve the following (**2.5** marks each) : 10

(*a*) If l, m, n are direction cosines of a line, then prove that $-l, -m, -n$ are also the direction cosines of that line.

(*b*) Write unsymmetrical form of equations of line and explain the terms involved.

(*c*) If $l_1, m_1, n_1; l_2, m_2, n_2; l_3, m_3, n_3$ are direction cosines of 3 mutually perpendicular lines, then write any **3** relations between these direction cosines.

(*d*) Write the characteristics of equation of a sphere.

P.T.O.

2. (*a*) Prove that every equation of first degree in x, y, z represents a plane. 6

(*b*) Find the angle between the pair of planes $3x - 4y + 5z = 0$ and $2x - y - 2z = 5$. 4

3. (*a*) Prove that the length of perpendicular from a point (x_1, y_1, z_1) to a line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ is square root of $(x_1 - \alpha)^2 + (y_1 - \beta)^2 + (z_1 - \gamma)^2 - [l(x_1 - \alpha) + m(y_1 - \beta) + n(z_1 - \gamma)]^2$. 6

(*b*) Obtain the symmetrical form the equations of the line $x - 2y + 3z = 4$, $2x - 3y + 4z = 5$. 4

4. (*a*) If the directions of the axes are changed without changing the origin, so that $l_1, m_1, n_1; l_2, m_2, n_2; l_3, m_3, n_3$ are the respective direction cosines of new axes OX', OY', OZ', then prove that the relation between the original co-ordinates (x, y, z) and new co-ordinates (x', y', z') of a point P are given by $x = l_1x' + l_2y' + l_3z'$, $y = m_1x' + m_2y' + m_3z'$, $z = n_1x' + n_2y' + n_3z'$. 6

(*b*) Find the equation of the plane $2x + 3y + 4z = 7$ referred to the point $(2, -3, 4)$ as origin, directions of the axes remaining the same. 4

5. (*a*) Prove that the locus of intersection of a sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ and a line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ is a pair of two points $(\alpha + lr_1, \beta + mr_1, \gamma + nr_1)$ and $(\alpha + lr_2, \beta + mr_2, \gamma + nr_2)$. 6

(*b*) Find the equation to the sphere through four points $(4, -1, 2)$, $(0, -2, 3)$, $(1, -5, -1)$, $(2, 0, 1)$. 4

6. Answer any *two* of the following (**5** marks each) : 10

(*a*) Show that the origin and the point (2, –4, 3) lie on different sides of the plane $x + 3y - 5z + 7 = 0$.

(*b*) Show that the line $\frac{1}{3}(x-2) = \frac{1}{4}(y-3) = \frac{1}{5}(z-4)$ is parallel to the plane $2x + y - 2z = 3$.

(*c*) If the origin O is changed to $O'(f, g, h)$ without changing the directions of the co-ordinate axes, then prove that the relation between the original co-ordinates (x, y, z) and new co-ordinates (x', y', z') of a point P are given by $x = x' + f, y = y' + g, z = z' + h$.

(*d*) Find the equation of tangent plane to the sphere $3(x^2 + y^2 + z^2) - 2x - 3y - 4z - 22 = 0$ at the point (1, 2, 3).