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SA—26—2025

FACULTY OF SCIENCE

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

APRIL/MAY, 2025

MATHEMATICS

(Complex Analysis-XV)

(Tuesday, 15-4-2025) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. Explain the method to obtain nth root of complex number $z = re^{iv}$. Hence find square root of $2i$. 15

Or

(*a*) Show that $u(x, y)$ is harmonic in some domain and find harmonic conjugate $v(x, y)$ when $u(x, y) = 2x(1 - y)$. 8

P.T.O.

(*b*) Prove that if a function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then its component functions u and v are harmonic in D. 7

2. Prove that any polynomial $P(z) = a_0 + a_1 z + a_2 z^2 + ... + a_n z^n$ where $a_n \neq 0$ of degree $n(n \geq 1)$ has at least one zero. 15

Or

(*a*) Let $w(t)$ be a continuous complex-valued function of t defined on an interval $a \leq t \leq b$. Then show that it is not necessarily true that there is a number C in the interval $a < t < b$ such that : 8

$$\int_a^b w(t)dt = w(c)(b-a)$$

(*b*) Let $w(t)$ be a piece-wise continuous complex-valued function defined on an interval $a \leq t \leq b$, then prove that : 7

$$\left|\int_a^b w(t)dt\right| \leq \int_a^b |w(t)|\, dt$$

3. Attempt any *two* of the following :

(*a*) Show that when a limit of a function $f(z)$ exists at a point z_0, then it is unique. 5

(*b*) If $f'(z) = 0$ everywhere in a domain D, then prove that $f(z)$ must be constant throughout D. 5

(*c*) Let C be the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant. Then show that : 5

$$\left|\int_C \frac{z+4}{z^3-1}\,dz\right| \le \frac{6\pi}{7}$$

(*d*) Let C be the positively oriented unit circle $|z| = 1$, then evaluate : 5

$$\int_C \frac{e^{2z}}{z^2}\,dz$$