This question paper contains **2** printed pages]

SA—41—2025

FACULTY OF SCIENCE AND TECHNOLOGY

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

APRIL/MAY, 2025

MATHEMATICS

Paper–XVI

(Integral Transforms)

(Thursday, 17-4-2025) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. If $L[f(t)] = F(s)$, then prove that :

$$L\left[\frac{1}{t} f(t)\right] = \int_s^\infty F(s)ds.$$

Hence find the Laplace transform of $\frac{\sin 2t}{t}$. 15

Or

(*a*) Using convolution theorem, find : 8

$$L^{-1}\left\{\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right\} a \neq b.$$

P.T.O.

(*b*) Obtain $L^{-1}\left(\frac{1}{s(s^2 + a^2)}\right)$. 7

2. Solve the initial value problem :

$$2y'' + 5y' + 2y = e^{-2t},\ y(0) = 1,\ y'(0) = 1.$$ 15

Or

Derive the Fourier sine and cosine integrals :

(*a*) $f(x) = \frac{2}{\pi}\int_0^\infty \sin ux\, du \int_0^\infty f(t) \sin ut\, dt$ 8

(*b*) $f(x) = \frac{2}{\pi}\int_0^\infty \sin ux\, du \int_0^\infty f(t) \cos ut\, dt$ 7

3. Attempt any *two* of the following : 5 each

(*a*) Prove that :

$L[f'(t)] = sL[f(t)] - f(0)$

where $L[f(t)] = F(s)$.

(*b*) Find the inverse Laplace transform of

$$\frac{s}{s^2 + 4s + 13}.$$

(*c*) Applying convolution solve the following initial value problem

$$y'' + y = \sin 3t$$

$y(0) = 0,\ y'(0) = 0.$

(*d*) Find the Fourier sine transform of $\frac{1}{x}$.