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SA—56—2025

FACULTY OF SCIENCE AND TECHNOLOGY

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

APRIL/MAY, 2025

MATHEMATICS

Paper XVII

(Topology)

(Monday, 21-4-2025) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (1) Attempt either (A) *or* (B) for Question Nos. **1** and **2**.

(2) All symbols carry usual meanings.

(3) Figures to the right indicate full marks.

1. (A) Define Topology, discrete topology, indiscrete topology and finite complement topology. Hence if X be a set. Let λ_f be the collection of all subsets U of X such that X-U either is finite or is all of X. Then show that λ_f is a topology on X. 15

P.T.O.

Or

(B) Attempt the following :

(*a*) Let X be an ordered set in the order topology; let Y be a subset of X that is convex in X. Then prove that the order topology on Y is the same as the topology Y inherits as a subspace of X. 8

(*b*) If β is a basis for the topology of X and C is a basis for the topology of Y, then the collection :

$$D = \{B \times C / B \in \beta \text{ and } C \in \mathbf{C}\}$$

is a basis for the topology of X × Y. 7

2. (A) Let X and Y be topological spaces, let $f : X \to Y$, then the following are equivalent : 15

(1) f is continuous

(2) For every subset A of X, one has $f(\bar{A}) \subset \overline{f(A)}$.

(3) For every closed set B of Y, the set $f^{-1}(B)$ is closed in X.

(4) For each $x \in X$ and each neighbourhood V of $f(x)$, there is a neighbourhood U of x such that $f(U) \subset V$.

Or

(B) Attempt the following :

(*a*) Let A be a subset of the topological space X; Let A′ be a set of all limit points of A. Then show that : 8

$$\bar{A} = A \cup A'.$$

(*b*) Prove that a finite Cartesian product of connected space is connected. 7

3. Attempt any *two* of the following : 5 each

(*a*) Let β and β' be basis for the topologies λ and λ', respectively on X. Then show that the following are equivalent :

(*i*) λ' is finer than λ.

(*ii*) For each $x \in X$ and each basis element $B \in \beta$ containing x, there is a basis element $B' \in \beta'$ such that $x \in B' \subset B$.

(*b*) If β is a basis for the topology of X, then show that the collection :

$$\beta_y = \{B \cap Y / B \in \beta\}$$

is a basis for the subspace topology on Y.

(*c*) Show that product of two Hausdorff space is Hausdorff.

(*d*) Show that the union of a collection of connected subspace of X that have a point in common is connected.