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SA—58—2025

FACULTY OF SCIENCE AND TECHNOLOGY

B.Sc. (Third Year) (Sixth Semester) EXAMINATION

APRIL/MAY, 2025

(New/CBCS Pattern)

MATHEMATICS

Paper XVII

(Elementary Number Theory)

(Monday-, 21-4-2025) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. (*a*) Let S be a set of positive integers with the following properties : 15

(*i*) The integer 1 belongs to S

(*ii*) Whenever the integer k is in S, the next integer $k + 1$ must also be in S. Prove that S is the set of all positive integers.

(*b*) Prove that for given integers a and b, not both of which are zero, there exist integers x and y such that $\gcd(a, b) = ax + by$.

Or

(*i*) If p is a prime and $p \mid a_1 a_2 \ldots \ldots \ldots . a_n$ then prove that $p \mid a_k$ for some k, $1 \leq k \leq n$. 8

(*ii*) Show that there are infinite number of primes. 7

P.T.O.

2. Prove that : 15

(*i*) Let $n_1, n_2, \dots \dots \dots, n_r$ be positive integers such that $\gcd(n_i, n_j) = 1$ For $i \neq j$, then the system of linear $x \equiv a_1 \pmod{n_1}$, $x \equiv a_2 \pmod{n_2}$ ... ... $x = a_r \pmod{n_r}$ has a simultaneous solution, which is unique modulo the integer $n_1.n_2 \dots \dots \dots n_r$.

(*ii*) If p is a prime then $(p-1)! \equiv -1 \pmod{p}$.

Or

(*a*) Prove that : 8

(*i*) If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then $a + c \equiv b + d \pmod{n}$

(*ii*) If $a \equiv b \pmod{n}$ then $a^k \equiv b^k \pmod{n}$ for any positive integer k.

(*b*) Show that there are an infinite number of primes of the form $4n + 3$. 7

3. Attempt any *two* of the following : 10

(*a*) If $a = qb + r$ prove that $\gcd(a, b) = \gcd(b, r)$.

(*b*) Find the values of x and y which satisfy the equation $56x + 72y = 40$.

(*c*) Let $p(x) = \sum_{k=0}^{m} c_k . x^k$ be a polynomial function of x with integral coefficient c_k, show that if $a \equiv b \pmod{n}$, then $p(a) \equiv p(b) \pmod{n}$.

(*d*) Show that $2^{340} \equiv 1 \pmod{341}$.