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SA—59—2025

FACULTY OF ARTS/SCIENCE

B.Sc. (Second Year) (Third Semester) EXAMINATION

APRIL/MAY, 2025

(New/CBCS Pattern)

MATHEMATICS

Paper-VI

(Real Analysis-I)

(Monday-, 21-4-2025) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

1. (*a*) Prove that, the intersection of arbitrary family of closed set is closed. Also show that the $\sqrt{8}$ is not rational number. 15

Or

(*b*) If $\{a_n\}$, $\{b_n\}$ be two sequences such that $\lim_{n\to\infty} a_n = a, \lim_{n\to\infty} b_n = b$, then prove that $\lim_{n\to\infty}(a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n = a + b$. 8

(*c*) Show that the sequence $\{S_n\}$, where $S_n = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{n+n}$, $\forall n \in N$ is convergent. 7

P.T.O.

2. (*a*) Prove that a necessary condition for convergence of an infinite series $\sum_{n=1}^{\infty} u_n$ is that $\lim_{n\to\infty} u_n = 0$. Hence show that the series $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}\ldots\ldots$ is not convergent. 15

Or

(*b*) If $\sum_{n=1}^{\infty} u_n$ is a positive term series such that $\lim_{n\to\infty} n\left(\frac{u_{n+1}}{u_n}-1\right)=l$ then prove that the series :

(*i*) converges, if $l > 1$,

(*ii*) diverges, if $l < 1$,

(*iii*) the test fails, if $l = 1$. 8

(*c*) Show that the series $\sum_{n=1}^{\infty} \frac{3.6.9\ldots 3n}{7.10.13\ldots(3n+4)} x^n, x > 0.$ converges if $x \leq 1$ and diverges if $x > 1$. 7

3. Attempt any *two* of the following : 10

(*a*) If S and T to subsets of real number then show that, $(S \cap T)' \Rightarrow S' \cap T'$

(*b*) If $\{a_n\}$, $\{b_n\}$ are two sequences such that :

(*i*) $a_n \leq b_n, \forall\ n,$

(*ii*) $\lim_{n\to\infty} a_n = a, \lim_{n\to\infty} b_n = b,$ then prove that $a \leq b$.

(*c*) Prove that, every absolute convergent series is convergent.

(*d*) Show that, the series $\frac{1.2}{3^2.4^2}+\frac{3.4}{5^2.6^2}+\frac{5.6}{7^2.8^2}+\ldots$ is convergent.