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SA—63—2025

FACULTY OF SCIENCE AND TECHNOLOGY

B.Sc. (Third Year) (Fifth Semester) EXAMINATION

APRIL/MAY, 2025

(CBCS New Pattern)

MATHEMATICS

Paper XIII

(Linear Algebra)

(Tuesday, 22-4-2025)   **Time : 10.00 a.m. to 12.00 noon**

Time—Two Hours   *Maximum Marks*—40

Note :—   (*i*)   *All* questions are compulsory.

   (*ii*)   Figures to the right indicate full marks.

1.   Let U and W be two subspaces of a vector space V and Z = U + W, then
     prove that Z = U $\oplus$ W iff the following condition is satisfied.
     Any vector **Z** $\in$ Z can be expressed uniquely as the :   15

$$\text{sum } z = u + w, \; u \in U, \; w \in W$$

Or

(*a*)   State and prove Rank-Nullity theorem.   8

(*b*)   Prove that in an *n*-dimensional vector space V any set of *n*-linearly
        independent vectors is basis.   7

P.T.O.

2. If S, T $\in$ L (U, V) and α, β are two scalars and : 15

$$\alpha\,(S + T) = \alpha S + \alpha T$$
$$(\alpha + \beta)\,S = \alpha S + \beta S$$

then prove that L(U, V) is vector space.

Or

(*a*) Define an inner product space. Also if V is an inner product space $u, v, w \in$ V and 'α' is scalar, then prove the following : 8

(*a*) $(u + v)\,.\,w = u\,.\,w + v\,.\,w$

(*b*) $u\,(\alpha\,v) = \bar{\alpha}\,(u.v)$

(*c*) $0\,.\,u = 0 = u\,.\,0$.

(*b*) Find the Eigen values of matrix : 7

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

3. Attempt any *two* of the following : 10

(*i*) If U and W are subspaces of a vector space V, then prove that :

(*a*) U $\cap$ W is a subspace of W

(*b*) U $\cup$ W is a subspace of V

Iff V $\subset$ W or W $\subseteq$ U

(*ii*) Let T : U $\rightarrow$ V be a linear map, then prove the following :

(*a*) R(T) is subspace of V

(*b*) N(T) is subspace of U.

(*iii*) If ST is non-singular, then prove that T is one-one and S is onto.

(*iv*) If T : $V_3 \rightarrow V_4$ be linear map defined by :

$T(u_1) = v_1 - 2v_2 + v_3 - v_4$

$T(u_2) = v_1 - v_2 + 2v_4$

$T(u_3) = 2v_2 + 3v_3 - v_4$

then find matrix of T w.r.t. standard basis.