This question paper contains **3** printed pages]

SA—68—2025

FACULTY OF SCIENCE AND ARTS

B.A./B.Sc. (Second Year) (Fourth Semester) EXAMINATION

APRIL/MAY, 2025

MATHEMATICS

Paper-IX

(Real Analysis-II)

(Tuesday, 22-4-2025) **Time : 2.00 p.m. to 4.00 p.m.**

Time—Two Hours *Maximum Marks*—40

N.B. :— (*i*) Attempt *all* questions.

(*ii*) Figures to the right indicate full marks.

1. (*i*) If a bounded function f is integrable on $[a, b]$, then prove that it is also integrable on $[a, c]$ and $[c, b]$, where c is a point of $[a, b]$. 15

(*ii*) If f is bounded and integrable on $[a, c]$, $[c, b]$, then prove that it is also integrable on $[a, b]$.

(*iii*) Prove that : 15

$$\int_a^b f dx = \int_a^c f dx + \int_c^b f dx, \quad a \le c \le b.$$

P.T.O.

Or

(*a*) If a function f is monotonic on $[a, b]$, then prove that it is integrable on $[a, b]$. 8

(*b*) If f is a non-negative continuous function on $[a, b]$ and $\int_a^b f\,dx = 0$, then prove that : 7

$$f(x) = 0, \text{ for all } x \in [a, b].$$

2. Prove that the improper integral $\int_a^b \frac{dx}{(x-a)^n}$ converges if and only if $n < 1$. Also, test the convergence of $\int_0^{\pi/2} \frac{\sin x}{x^p}\,dx$. 15

Or

(*a*) Show that the integral $\int_0^\infty x^{m-1} e^{-x}\,dx$ is convergent if and only if $m > 0$. 8

(*b*) If ϕ is bounded and monotonic in $[a, \infty)$ and tends to 0 as $x \to \infty$, and $\int_a^x f dx$ is bounded for $X \geq a$, then prove that $\int_a^\infty f\phi\,dx$ is convergent at ∞. 7

3. Attempt any *two* of the following : 10

(*a*) Show that a constant function k is integrable and $\int_a^b k\,dx = k(b-a)$.

(*b*) If f and g are integrable on $[a, b]$ and g keeps the same sign over $[a, b]$, then prove that there exists a number μ lying between the bounds of f such that :

$$\int_a^b fg\,dx = \mu \int_a^b g\,dx.$$

(*c*) If f and g be two functions such that $f(x) \le g(x)$, for all x in $[a, X]$, then prove that :

(*i*) $\int_a^\infty f\,dx$ converges, if $\int_a^\infty g\,dx$ converges,

(*ii*) $\int_a^\infty g\,dx$ converges, if $\int_a^\infty f\,dx$ converges.

(*d*) Examine the convergence of $\int_0^2 \frac{dx}{(2x - x^2)}$.