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SA—74—2025

FACULTY OF SCIENCE

B.Sc. (Second Year) (Third Semester) EXAMINATION

APRIL/MAY, 2025

(CBCS/New Pattern)

MATHEMATICS

Paper VII

(Group Theory)

(Wednesday, 23-4-2025)   **Time : 2.00 p.m. to 4.00 p.m.**

Time—Two Hours   *Maximum Marks*—40

Note :—   (*i*)   Attempt *all* questions.

  (*ii*)   Illustrate your answer with suitably labelled diagrams wherever necessary.

  (*iii*)   Figures to the right indicate full marks.

1.   If A be a non-empty set and let R be an equivalence relation in A. Let a and b be arbitrary elements in A, then prove that :   15

  (*i*) $a \in [a]$

  (*ii*)   If $b \in [a]$, then $[b] = [a]$

  (*iii*) $[a] = [b]$ iff $(a, b) \in R$

  (*iv*)   Either $[a] = [b]$ *or* $[a] \wedge [b] = \phi$.

P.T.O.

Or

(*a*) State and prove Lagrange's theorem. 8

(*b*) Prove that every group of prime order is cyclic. 7

2. Prove that the set of residue classes modulo m is an abelian group of order m with respect to addition of residue classes. 15

Or

(*a*) Suppose G is a group and N is a normal subgroup of G. Let f be a mapping from G to G|N defined by :

$$f(x) = N.x \ \forall \ x \in G$$

then prove that f is a homomorphism of G onto G|N and kernel of f is N. 8

(*b*) Prove that intersection of any collection of normal subgroup is itself a normal subgroup. 7

3. Attempt any *two* of the following : 5 each

(*a*) If H is any subgroup of G and $h \in H$, then prove that $Hh = H = hH$.

(*b*) Let a be a fixed element of a group G, then prove that the mapping $f_a : G \to G$ defined by :

$$f_a(x) = a^{-1} xa \ \forall \ x \in G$$

is an automorphism of G.

(*c*) If I be the set of all integers and R be a relation in I defined as $x\mathrm{R}_y$ holds iff $x - y$ is divisible by 5, $x, y \in$ I, then prove that R is an equivalence relation.

(*d*) Prepare the composition table for the group :

$$\mathrm{G} = \{0,\ 1,\ 2,\ 3,\ 4\}$$

with respect to addition modulo 5 and write down the inverse of each element of G.