This question paper contains **4** printed pages]

SA—78—2025

FACULTY OF SCIENCE

B.Sc. (Third Year) (Fifth Semester) EXAMINATION

APRIL/MAY, 2025

(CBCS/New Pattern)

MATHEMATICS

Paper XIV

(Operation Research)

(Thursday, 24-4-2025) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (1) *All* questions are compulsory.

(2) Figures to the right indicate full marks.

1. Explain the three components of linear programming problem. Also, the manager of an oil refinery must decide on the optimum mix of two possible blending processes of which the input and output production runs are as follows : 15

Process	**Input**		**Output**	
	Crude A	**Crude B**	**Gasoline X**	**Gasoline Y**
1	6	4	6	9
2	5	6	5	5

P.T.O.

The maximum amounts available of crudes A and B are 250 units and 200 units respectively. Market demand shows that at least 150 units of gasoline X and 130 units of gasoline Y must be produced. The profit per production run from process 1 and process 2 are Rs. 4 and Rs. 5 respectively. Formulate the problem for maximising the profit.

Or

(*a*) What are the major steps in the solution of a Linear programming problem by graphical method ? 7

(*b*) Using graphical method, solve the L.P.P. : 8

Maximize : $Z = 2x_1 + 4x_2$

Subject to the constraints :

$$x_1 + 2x_2 \leq 5,$$

$$x_1 + x_2 \leq 5$$

and $x_1, x_2 \geq 0.$

2. Prove that, if an L.P.P. has a feasible solution, then it also has a basic feasible solution. 15

Or

(*a*) A student has to select one and only one elective in each semester and the same elective should not be selected in different semesters. Due

to various reasons, the expected grades in each subject, if selected in different semester, vary and they are given below : 7

Semester	Analysis	Statistics	Graph Theory	Algebra
I	F	E	D	C
II	E	E	C	C
III	C	D	C	A
IV	B	A	H	H

The grade points are : H = 10, A = 9, B = 8, C = 7, D = 6, E = 5 and F = 4. How will the student select the elective in order to maximize the total expected points and what will be his maximum expected total points ?

(*b*) A company wishes to assign 3 jobs to 3 machines in such a way that each job is assigned to some machine and no machine works on more than one job. The cost of assigning job i to machine j is given by the matrix below (ijth entry) : 8

$$\text{Cost matrix : } \begin{bmatrix} 8 & 7 & 6 \\ 5 & 7 & 8 \\ 6 & 8 & 7 \end{bmatrix}$$

Draw the associated network and also formulate the network L.P.P.

3. Attempt any *two* of the following : 5 each

(*a*) Prove that, a necessary and sufficient condition for the existence of a feasible solution to the general transportation problem is that :

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \lambda(\text{say}).$$

P.T.O.

(*b*) Explain the iterative procedure of Big-M method of L.P.P.

(*c*) Rewrite in standard form the following L.P.P. :

Minimize : $Z = 2x_1 + x_2 + 4x_3$

Subject to the constraints :

$$-2x_1 + 4x_2 \leq 4,$$

$$x_1 + 2x_2 + x_3 \geq 5,$$

$$2x_1 + 3x_2 \leq 2,$$

$$x_1, x_2 \geq 0.$$

and x_3 unrestricted in sign.

(*d*) Three grades of coal A, B and C contain ash and phosphorus as impurities. In a particular industrial process a fuel obtained by blending the above grades containing not more than 25% ash and 0.03% phosphorus is required. The maximum demand of the fuel is 100 tons. Percentage impurities and costs of the various grades of coal are shown below. Assuming that there is an unlimited supply of each of coal and there is no loss in blending, formulate the blending problem to minimize the cost :

Coal grade	**% ash**	**% phosphorus**	**Cost per ton** (in Rs.)
A	30	0.02	240
B	20	0.04	300
C	35	0.03	280